Central bank collateral asset fire sales regulation and liquidity

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This paper analyses the potential roles of bank asset fire sales and recourse to central bank credit to ensure stability of banks' short term bank liabilities. Two causes of bank default are distinguished, namely illiquidity and insolvency. Short term funding stability is modeled as a strategic game between short term depositors who decide to withdraw their deposits or not. The liquidity properties of bank assets are stylized as exponential function. An optimal liability structure of banks is derived depending on asset liquidity, the central bank collateral framework, and regulation. Various effects are identified that shed new light on the way these three factors shape the ability of the banking system to deliver both on maturity transformation and on financial stability. The model also allows for a better understanding of why a sudden non-anticipated reduction of asset liquidity (or tightening of the collateral framework) can destabilize short term liabilities of banks and more generally how the collateral framework can be understood, beyond its essential aim to protect the central bank, as financial stability and non-conventional monetary policy instrument (the latter in particular at the zero lower bound).

JEL codes: E42, G21
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1. Introduction

The model proposed in this paper sheds new light on how asset liquidity, Basel III type liquidity regulation, and the central bank collateral framework affect financial stability and monetary policy. In January 2013, the Basle Committee on Banking Supervision has issued again, after a number of adaptations, a document describing **Basle III liquidity regulation**, and in particular the so-called Liquidity Coverage Ratio (LCR) and the related concept of High Quality Liquid Assets (HQLA). The Financial crisis of 2007/2008 is said to also have been triggered by the insufficient asset liquidity buffers of banks relative to their short term liabilities. These insufficient buffers would have led to an (at least temporarily) excessive reliance on central bank funding. In the words of Basel Committee (2013):

> *2. During the early "liquidity phase" of the financial crisis that began in 2007, many banks – despite adequate capital levels – still experienced difficulties because they did not manage their liquidity in a prudent manner. The crisis drove home the importance of liquidity to the proper functioning of financial markets and the banking sector. Prior to the crisis, asset markets were buoyant and funding was readily available at low cost. The rapid reversal in market conditions illustrated how quickly liquidity can evaporate, and that illiquidity can last for an extended period of time. The banking system came under severe stress, which necessitated central bank action to support both the functioning of money markets and, in some cases, individual institutions.*

This excessive reliance on central bank funding as alluded to in the last sentence is considered to have constituted a form of moral hazard. The LCR requires a certain amount of HQLAs to be maintained by banks relative to possible liquidity outflows in a one month stress scenario. The Basle Committee (2013) considers as constituting characteristic of HQLAs that they can be fire-sold without large losses even under stressed circumstances:

> *24. Assets are considered to be HQLA if they can be easily and immediately converted into cash at little or no loss of value. The liquidity of an asset depends on the underlying stress scenario, the volume to be monetised and the timeframe considered. Nevertheless, there are certain assets that are more likely to generate funds without incurring large discounts in sale or repurchase agreement (repo) markets due to fire-sales even in times of stress*

As recalled by e.g. Brunnermeier et al (2009), negative externalities are key to justify regulation of financial markets. In case of liquidity regulation, these negative externalities relate in particular to the asset fire sales spiral and more generally to various forms of negative contagion. According to the Financial Services Authority (2009, 68), "liquidity risk has inherently systemic characteristics, with the reaction of one bank to liquidity strains capable of creating major liquidity strains for others." Also the Basel Committee (2013) refers to the negative externalities of asset fire sales:

> *25. … An attempt by a bank to raise liquidity from lower quality assets under conditions of severe market stress would entail acceptance of a large fire-sale discount or haircut to compensate for high market risk. That may not only erode the market's confidence in the bank, but would also generate mark-to-market losses for banks holding similar instruments and add to the pressure on their liquidity position, thus encouraging further fire sales and declines in prices and market liquidity.*

Finally, the welfare economic tradeoffs between the efficiency of the banking system in delivering maturity transformation and financial stability is eventually crucial when assessing the net benefits of regulation for society. In the words of the Turner review (Services Authority, 2009, 68):

> *[T]here is a tradeoff to be struck. Increased maturity transformation delivers benefits to the non bank sectors of the economy and produces term structures of interest rates more favourable to long-term investment. But the greater the aggregate degree of maturity transformation, the more the systemic risks and the greater the extent to which risks can only be offset by the potential for central bank liquidity assistance.*

This trade-off, and the revealed reservations of regulators regarding large central bank reliance of banks under liquidity stress, and at the same time the regulators awareness that asset fire sales of imperfectly liquid assets create negative externalities (that could be avoided by the recourse to central bank credit), suggests that analyzing the impact of liquidity regulation must integrate the role of central bank funding.

While the central bank collateral framework got relatively limited attention in academic writing, it is one of the most complex and economically significant elements of monetary policy implementation. Unencumbered central bank eligible collateral is potential liquidity, as it can, in principle, be swapped into central bank money. It is therefore not exaggerated to argue that the collateral framework must be an important ingredient of any theory of liquidity crises (as noted by Bagehot, 1873), and of any monetary theory. A survey of current central bank practice in G20 countries is provided by Markets Committee (2013). Section 1.3 of this report also summarizes the various measures taken during the financial crisis by central banks (p 8-9):

> *"During the height of the financial crisis in 2008–09, a number of central banks introduced, to varying degrees, crisis management measures such as a temporary acceptance of additional types of collateral, a temporary lowering of the minimum rating requirements of existing eligible collateral or a temporary relaxation of haircut standards. Many of these temporary changes have expired."*

Changes to the collateral framework also seem to play a role as monetary policy instrument (i.e. beyond being an instrument to address a liquidity crisis) when central banks approach the zero lower interest rate bound. For example, the Bank of England's "Funding for lending" (FLS) scheme also relies on a widening of the collateral framework (see Bank of England, 2012, 317).

> *"A broad range of collateral is eligible for use in the FLS, so that, as far as possible, the availability of collateral does not constrain banks' ability to use the FLS. Therefore eligible collateral in the FLS… includes portfolios of loans, various forms of asset-backed securities and covered bonds, and sovereign and central bank debt."*

Portfolios of loans and ABS are not eligible for ordinary Bank of England credit open market operations.

So far, the **literature** on the interaction between central bank borrowing against eligible collateral, asset fire sales, and liquidity regulation is relatively limited (e.g. Bindseil and Lamoot, 2011, Bech and Keister, 2012, ECB, 2013, Körding and Scheubel, 2013, Bindseil and Jablecki, 2013). This paper provides a first model which allows understanding the roles of the two emergency liquidity sources in equilibrium, and how changes of (i) asset liquidity; (ii) the central bank collateral framework; and (iii) regulation impact on this equilibrium. The paper also revisits the merits and channels of effectiveness of liquidity regulation and collateral policies for financial stability and monetary policy.

2. Funding stability – a model with continuous asset liquidity and haircuts

Throughout this note, we consider the following stylized bank balance sheet. The total length of the balance sheet has been set to unity. Assets are heterogeneous in a continuous sense, while there are three types of liabilities which are each separately homogenous (with $e \in [0,1], t \in [0,1], e+t \in [0,1]$)

Figure 1: A stylised bank balance sheet to analyse liquidity uncertainty

Assets		Liabilities	
Assets	1	Short term debt 1	(1-t-e)/2
		Short term debt 2	(1-t-e)/2
		Long term debt ("term funding")	t
		Equity	e

The stylized balance sheet is sufficient to capture one key issue of banking: how to ensure the confidence of short term creditors of the bank such that they do not easily switch to a fear mode in which they start withdrawing deposits, triggering self-fulfilling destructive dynamics ending in bank default. This is the well-known bank run problem, as analyzed for instance by Diamond and Dybvig (1982). Confidence may be sustained in particular by two means. *First*, the bank may limit the role of short term funding. However, in general investors prefer to hold short term debt instruments over long term debt instruments, and hence require a higher interest rate on long term debt. In other words, long term debt is associated with higher funding costs for the bank, or, put differently, maturity transformation is one of the key contributions of banking to society (see Financial Services Authority, 2009, 68). *Second*, the bank may aim at holding sufficient amounts of liquid assets, both in the sense of being able to liquidate these assets in case of need, and in order to be able to pledge them with the central bank at limited haircuts. However, on average, liquid assets generate lower return than illiquid ones (e.g. Houvelinga 2005, or Chen et al, 2007 for recent empirical studies). Consider now in more detail the different balance sheet positions.

Bank assets

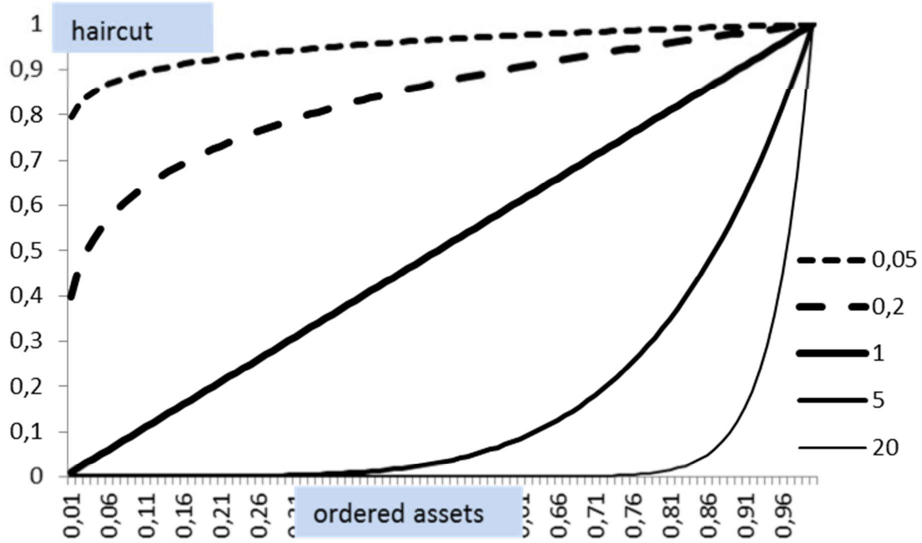
The total assets of the banks have initial value 1. Assets are not homogeneous, but can be differentiated as follows: (i) Asset liquidity, as measured by the "fire sale" discount to be accepted if an asset is to be sold in the short run; (ii) Eligibility and haircut if submitted as central bank collateral. Assets are either central bank eligible or ineligible, and if they are eligible they are accepted at a certain haircut. This is simplified in the model to the assumption that all assets are eligible, but have varying haircuts between 0 and 1; (iii) Finally, assets have a differentiated treatment in liquidity regulation, as they are accepted or not as "high quality liquid asset" (HQLA) (see annex 4 of Basle Committee, 2013, for a summary of the envisaged standard treatment).

Assume that there is a continuum of assets and that assets are ranked from those which the central bank considers the best quality collateral to the ones that it considers the least suitable collateral. The **central bank collateral haircut function** is then a function from the assets unity space [0,1] into the possible haircuts [0,1]. Assume that it has the following functional form with δ≥0:

$$h(x)=x^{\delta} \quad (1)$$

If δ is close to 0, then the haircuts increase and converge quickly towards 1. If in contrast δ is large (say 10) then haircuts stay at close to zero for a while and only start to increase in a convex manner when approaching the least liquid assets. The total haircut (and the average haircut) if all assets are pledged is $1/(\delta+1)$, and hence potential central bank credit is $\delta/(\delta+1)$. This is obtained from the integration rule $\int x^{\delta} = x^{\delta+1}/(\delta+1)$. The following figure illustrates the haircut function h(x) for various values of δ.

Figure 2: the exponential haircut function $h(x)=x^{\delta}$ in the unity space for different values of δ



For example, in the case of the Eurosystem, out of EUR 32 trillion of aggregated bank assets, around EUR 7-8 trillion are eligible central bank collateral, which may account for around EUR 5-6 trillion after haircuts. The eligibility criteria and haircut matrices are provided in ECB (2012) and one can match this information in principle with an informed guess of banks' assets holdings. This implies that the average haircut applied by the Eurosystem to (the entirety of) bank assets is around 84% (27 trillion /32 trillion), and central bank refinancing power is 16% of eligible assets, which approximately implies a parameter value δ=0.2.

Now consider **asset liquidity** in the sense of the ability of banks to sell assets in the short term horizon without this inflicting value losses and hence a loss for the bank. Assume again that assets are ranked from the most liquid to the least liquid, and that the fire sale discount function is a function from [0,1] into [0,1] with the following function form, with θ≥0.

$$d(x)=x^{\theta} \quad (2)$$

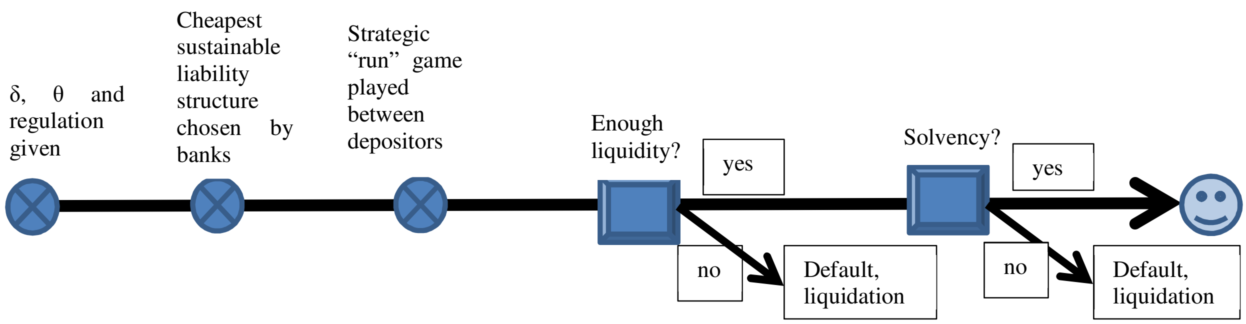
If θ is close to 0, then the fire sale discounts increase and converge quickly towards 1. If in contrast θ is large (say 10) then discounts are close to zero for most assets and only start to increase in a convex manner when approaching the least liquid assets. If a certain share x of the assets has to be sold, then the total fire sale discount eating up equity will be, assuming that the bank starts with the most liquid assets: $x^{\theta+1}/(\theta+1)$. Empirical estimates of default costs in the corporate finance literature vary between 10 and 44% (see e.g. Glover, 2011, and Davydenko et al 2011). In fact this cost can be based on the liquidation cost of assets, captured in the parameter θ. Liquidation of all assets will lead to a damage of $1/(1+\theta)$, such that remaining asset value will be $\theta/(1+\theta)$. If default cost is 10%, this would mean that θ = 9, and if default cost is 44%, then θ = 1.27. For a value of default costs in the middle of the empirical estimates of say 25%, one obtains θ=3.

Bank liabilities
Short term liabilities are equally split to two ex-ante identical depositors. The analysis will identify under which circumstances there will be no incentive to "run" in a simple strategic game of the two depositors. (ii) Long term debt does not mature within the period considered. It ranks pari passu with short term debt. (iii) Equity is junior to all other liabilities, and cannot flow out either. It is assumed that central bank borrowing is zero initially. However, it can substitute for outflows of short term liabilities in case of need. It is collateralized and therefore the central bank acquires in case of default ownership of the assets pledged as collateral. Apart from this, the central bank claim ranks pari passu, i.e. remaining claims after collateral liquidation are treated in the same way as an initial unsecured deposit. The sum of liabilities obviously has to be one. In the balance sheet above, short term debt is presented as the residual item, but this is purely presentational (i.e. any other liability item could have been presented as the residual – this presentation serves the objective to make visible that the balance sheet identity is respected).

Time line
The model as presented in sections 2 to 5 is based on the following time line:
- Initially, the bank has the above shown balance sheet composition
- Short term depositors/investors play a strategic game with two alternative actions: to run or not to run. "Running" means to withdraw the deposits and to transfer them to another account. If successful, this means that the value of deposits is afterwards equal to the initial value minus a small cost capturing the transaction cost of withdrawing the deposits, which is ε.
- It is not to be taken for granted that depositors can withdraw all their funds. If one or both of the depositors run, then at least one, or several of the following will apply: (i) Substitution of deposit outflows with central bank credit, assuming that the bank has sufficient eligible collateral. (ii) Liquidation of assets: if central bank collateral is insufficient to completely substitute short term funding withdrawals, then the bank will also do asset fire sales. (iii) If it is impossible to pay out the depositors that want to withdraw their deposits, Illiquidity induced default will occur. If illiquidity induced default occurs, all (remaining) assets need to be liquidated, and corresponding default related losses occur.
- If the bank was not closed due to illiquidity in the previous stage, still its solvency is analyzed and if capital is negative, the bank is resolved. Again, it is assumed in this case that the full costs of immediately liquidating all assets materialize.

The full timeline is summarized in the following figure. Sections 3 – 5 take the first two nodes as given and start directly with the third, i.e. with the strategic game. Sections 6 and 7 work backwards to the beginning of this time line.

Figure 1: Time-line of bank liability structure model



3. Pure reliance on central bank funding

In this section it is assumed that asset liquidation is not an option, say because markets are totally frozen, i.e. θ=0. Therefore, the analysis can focus on the sufficiency or not of buffers for central bank credit. The following proposition states the necessary condition for funding stability of banks in this case.

***Proposition 1**: If $\theta = 0$, and assuming a small transaction cost ε of withdrawing deposits, a Strict Nash No-Run (SNNR) equilibrium prevails if and only if δ/(δ+1) ≥ (1-t-e)/2, i.e. the liquidity buffer based on recourse to the central bank is not smaller than one half of the short term deposits.*

The decision set of depositor i (i=1,2) from which he will choose his decision D_i consists in $\{K_i, R_i\}$, whereby "K" stands for "keeping" deposits and "R" stands for "run". Call the pay-off function of depositor i $U_i = U_i(D_1, D_2)$. Note that the strategic game is symmetric, i.e. $U_1(K_1, K_2) = U_2(K_1, K_2)$ $U_1(K_1, D_2) = U_2(D_1, K_2)$, $U_1(R_1, K_2) = U_2(K_1, R_2)$, $U_1(R_1, R_2) = U_2(R_1, R_2)$. This allows expressing in the rest of the paper conditions only with reference to one of the two players.

A *Strict Nash equilibrium* is defined as a strategic game in which each player has a unique best response to the other players' strategies (see Fudenberg and Tirole, 1991, 11). A *Strict Nash No-Run (SNNR) equilibrium* in the run game is therefore one in which the no-run choice dominates the "run" choice regardless of what the other depositors decide, i.e. an SNNR equilibrium is defined by

$$U_1(K_1, K_2) > U_1(R_1, K_2) \cap \; U_1(K_1, R_2) > U_1(R_1, R_2)$$

Apart from the following proof of Proposition 1, the proofs will be provided in annexes. To prove proposition 1 (and similar subsequent propositions), it is sufficient to calculate through the pay-offs under the possible parameter combinations and establish the frontiers of parameter combinations under which the conditions of an SNNR equilibrium apply.

For Proposition 1, Figures 3, 4 and 5 show the strategic game in the three possible cases (1-t-e) < δ/(δ+1), (1-t-e)/2 < δ/(δ+1) < (1-t-e), and δ/(δ+1)< (1-t-e)/2, respectively. Pay-off fields are shaded in grey if they reflect an outcome in which bank default occurs and the bank is liquidated. In the case assumed here that θ=0, a default means complete destruction of asset value as the liquidation value of assets is zero.

Figure 3: The bank run game if (1-t-e) < δ/(δ+1) and $\theta = 0, \varepsilon > 0$.

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε (1-t-e)/2
	Run	(1-t-e)/2 (1-t-e)/2 -ε	(1-t-e)/2-ε (1-t-e)/2-ε

Figure 4: The bank run game with (1-t-e)/2 < δ/(δ+1) < (1-t-e) and $\theta = 0$, $\varepsilon > 0$.

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε (1-t-e)/2
	Run	(1-t-e)/2 (1-t-e)/2 -ε	(δ /(δ+1))/2 (δ /(δ+1))/2

Figure 5: The bank run game if δ/(δ+1) < (1-t-e)/2 and $\theta = 0, \varepsilon > 0$.

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	δ/(δ+1) 0
	Run	0 δ/(δ+1)	(δ /(δ+1))/2 (δ /(δ+1))/2

The proposition follows from verifying the condition $U_1(K_1, K_2) > U_1(R_1, K_2)$ and $U_1(K_1, R_2) > U_1(R_1, R_2)$. It is apparent from figures 3 to 5 that the conditions are met only under the conditions as stated in proposition 1. □

The key problem with the case δ/(δ+1) < (1-t-e)/2 is that $U_1(K_1, R_2) > U_1(R_1, R_2)$ does not hold. If the other depositor runs, then it is better to run to get some money out of the bank before the bank defaults, i.e. it is better to get out (δ/(δ+1))/2 and thereby to contribute to the bank default than to let the other depositor get out δ/(δ+1), trigger bank default, and have a payoff of zero because the remaining liquidation value of the assets of the bank is zero.

Note that in the rest of the paper, the case ε>0 will always be assumed, i.e. that there is in any case a low but positive cost of withdrawing deposits. This is only for the purpose of simplicity, and it is possible to calculate through the model also for the case of absence of such a factor.

4. Pure reliance on asset fire sales

Now consider the case in which the central bank accepts no collateral at all (or just does not offer any credit operations with banks as it prefers to implement monetary policy exclusively through outright operations). In this case **δ=0, such that addressing a bank run will rely exclusively on asset liquidation**. Assume that the bank does whatever it takes in terms of asset liquidation to avoid disorderly illiquidity induced insolvency. The total amount of liquidity that the bank can generate through asset fire sales is θ/(θ+1). Therefore, illiquidity induced default will materialise only if deposit withdrawals eventually exceed this amount. While with full reliance on central bank lending, the question was whether the related liquidity buffers would be sufficient (and if not, who would recover what), in the present case, two default triggering events need to be considered. Indeed, even if the bank has survived a liquidity withdrawal, it may afterwards be assessed as insolvent and thus be liquidated at the request of the bank supervisor. As noted above, for a given liquidity withdrawal x, the fire sale related loss is $x^{\theta+1}/(\theta+1)$. The latter default event occurs if this loss exceeds equity.

Note that it is assumed that equity is never sufficient to absorb the losses resulting from a bank default, i.e. it is assumed that e ≤ 1/(θ+1). Of course one could also calculate through the excluded case, but it is omitted here as it does not seem to match real cases and to not multiply cases.

Proposition 2: If $\delta = 0$, a SNNR equilibrium exists iff (1-t-e)/2≤θ/(θ+1) and $e \geq ((1-t-e)/2)^{\theta+1}/(\theta+1)$

The proposition can be verified by again establishing the strategic game pay-offs and showing under which circumstances the conditions $U_1(K_1, K_2) > U_1(R_1, K_2)$ and $U_1(K_1, R_2) > U_1(R_1, R_2)$ are met. The proof is provided in annex 1. In sum, to ensure financial stability in the case of absence of central bank credit, minimum liquidity and capital buffers are needed in some appropriate combination to ensure the stability of a given amount of short term funding. The lower the asset liquidity, the lower the amount of short term funding that can be sustained for a given level of equity.

5. Cases in which the bank may want to rely on both types of liquidity buffers

Now consider the cases in which both **$\theta>0$ and $\delta>0$**. It is assumed that the ordering of assets is the same for both forms of liquidity generation, i.e. if asset i is subject to lower fire sale discounts than asset j, then also asset i will have a lower central bank collateral haircut than asset j.

Proposition 3 narrows down the actual range of mixed cases, i.e. cases in which both liquidity sources play a role in the planning of the bank.

***Proposition 3**: if either $\delta \geq \theta$ or ($\theta > \delta \cap \delta/(\delta+1) \geq (1-t-e)/2$), then banks will only rely on central bank credit to address possible deposit withdrawals, and hence the conditions established in proposition 1 apply to the existence of an SNNR equilibrium.*

Taking recourse to the central bank does not cause a loss, while fire sales cause one. If in addition, central bank recourse yields more liquidity (as by assumption $\delta>\theta$), then central bank credit strictly dominates asset fire sales as a source of emergency liquidity. If $\theta>\delta$ and central bank liquidity buffers allow to address liquidity outflows relating to one depositor, i.e. $\delta/(\delta+1) \geq (1-t-e)/2$, which, as shown previously, allows to sustain the SNNR equilibrium, then again relying only on central bank credit dominates strategies to rely on both sources. □

The cases in which the bank wants to rely potentially **on both funding sources** therefore appear to be limited to the ones in which $\theta>\delta$ and $(1-t-e)/2>\delta/(\delta+1)$. Again a number of cases have to be distinguished. There will generally be a trade-off between the maximum liquidity generation and the ability to avoid losses, under the optimal use of the two funding sources. For example, the maximum generation of liquidity is achieved through fire sales only, and will be equal to $\theta/(1+\theta)$. However, this also leads to the highest possible fire sales losses $1/(1+\theta)$, and it is realistic to assume that this extent of losses would exceed equity, and anyway if all of the assets of the bank are sold, it has ceased to exist. The lowest generation of liquidity is achieved if all assets are pledged through central bank credit, and in this case liquidity generation is $\delta/(1+\delta)$ and fire sale losses are 0. Between these extreme points of pairs (liquidity generation, fire sale losses), the set of efficient combinations of the two variables can be calculated. The following proposition addresses the question whether the bank's strategy should foresee to fire sale the most liquid assets and pledge the rest with the central bank, or the other way round.

***Proposition 4**: In funding strategies to address withdrawals of short term deposits relying on both funding sources, the bank should always foresee to fire sale the most liquid assets and pledge the rest as collateral with the central bank (and not the other way round).*

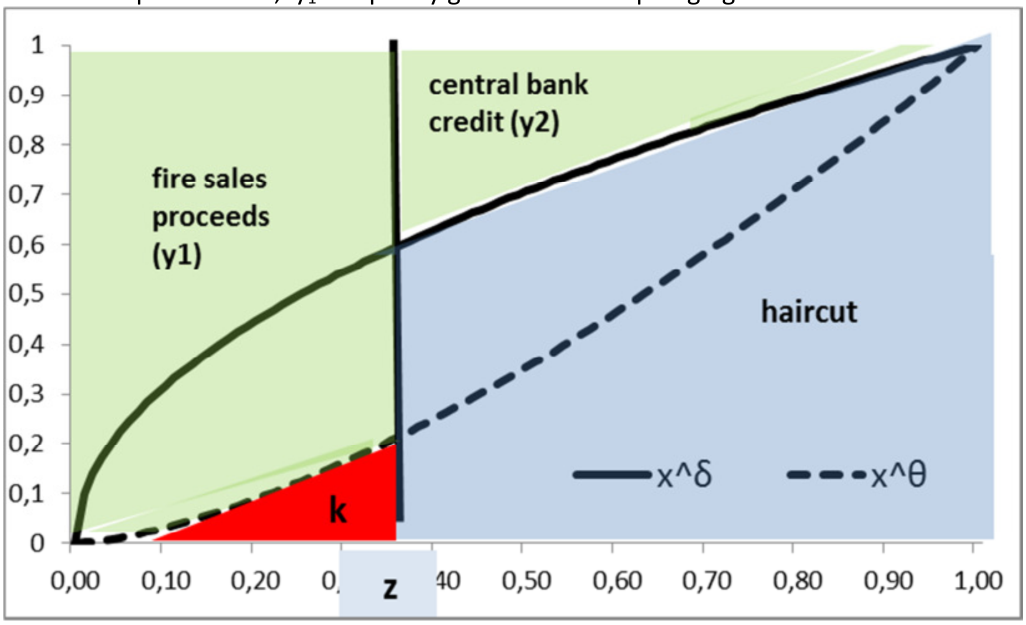
The proof of this proposition is provided in annex 2. The proof relies on showing that with the strategy to fire sale the most liquid assets and pledge the rest, the bank can achieve combinations of liquidity generation and fire sale cost, which are always superior to the combinations under the reverse strategy.

Proposition 5 provides the condition in the case of strategies relying on both funding sources for a SNNR equilibrium, depending on the initial liability structure of the bank and the parameters θ and δ.

Proposition 5: *Let* $z \in [0,1]$ *determine which share of its assets is foreseen by the bank to be used for fire sales (i.e. the less liquid share 1-z of assets are foreseen for pledging with the central bank). Let k=h(z) be the fire sales from fire selling the z most liquid assets and let y=f(z) be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets (1-z). Then a SNNR equilibrium exists if and only if* $\exists\, z \in [0,1]$: $y = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1} \geq (1-t-e)/2$ and $k = h(z) = \frac{z^{(\theta+1)}}{\theta+1} \leq e.$

The proof of proposition 3 is provided in annex 3. The following figure illustrates the generation of liquidity and fire sale losses under strategy z. Figure 6 assumes that the bank plans to fire sale the most liquid part of its assets z, and pledge with the central bank the least liquid part of assets (1-z). Therefore, total liquidity y that could be generated corresponds to the sum of y_2, the surface above the fire sale loss curve x^{θ} up to z, and y_2, the surface above the haircut curve x^{δ}, starting at z. Fire sale losses k will be equal to the surface below the fire sale loss curve between 0 and z.

Figure 6: Use of bank assets according to strategy z (y_2 is liquidity and k loss generated from fire-selling the most liquid assets z; y_1 is liquidity generated from pledging with central bank least liquid assets (1-z))



The following proposition 6 describes the nature of the combinations of y and k that can be achieved by varying z between 0 and 1. This proposition will be the basis for finding an optimal liability structure of the bank as discussed in the subsequent two sections. The optimal strategy z is determined by the idea to start from the least liquid assets and pledge with the central bank everything as collateral until one needs to switch in order to achieve the necessary total liquidity y. One should switch as late as possible such as to minimise fire sale losses. If one never switches then as it was assumed (to achieve a true mixed case in terms of emergency liquidity sources) that (1-t-e)/2 > δ/(δ+1) one will not get enough liquidity. If one switches too early one does not minimise fire sale losses and hence one needs more equity to sustain the strategy.

Proposition 6*: Let k=h(z) be the fire sales from fire selling the z most liquid assets and let y=f(z) be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets (1-z), whereby k(0)=0, k(1)=* 1/(1+θ) and *dk/dz >0 and y(0)=* $\delta/(\delta+1)$*, y(1)=* θ/(1+θ) and *dy/dz < 0.*

Then there is a non-decreasing function k=g(y): [0, θ/(1+θ)] → [0, 1/(1+θ)] *with*:

$$\forall\ y \in \left[0, \frac{\delta}{\delta+1}\right]: g(y) = 0\ ; \qquad \forall\ y \in \left[\frac{\delta}{\delta+1}, \frac{\theta}{1+\theta}\right]: \frac{dg}{dy} > 0; \qquad g\left(\frac{\theta}{1+\theta}\right) = \frac{1}{1+\theta}.$$

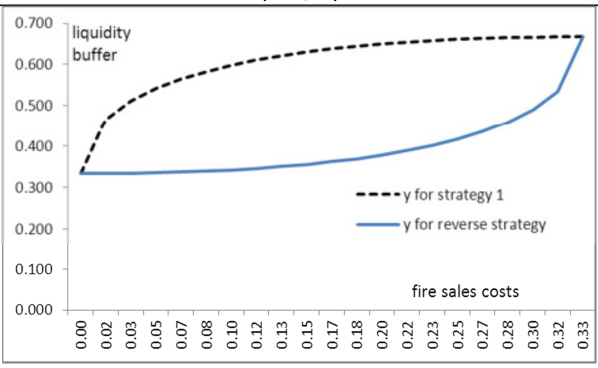
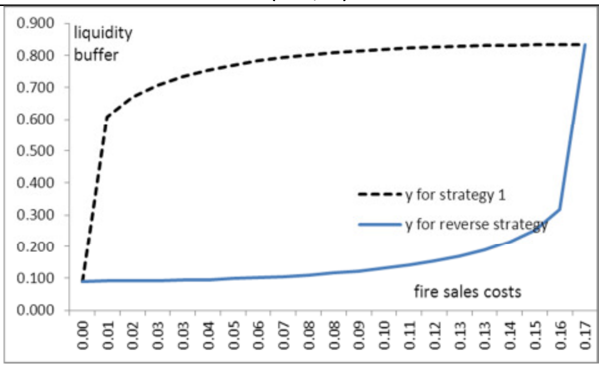
In the proof of proposition 5, it was shown that the total liquidity generated by strategy z was $y = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1}$. This is a monotonously declining function since the second term is smaller than the third term and the third term grows faster than the second term. It therefore can be inverted into the function $z=f^{-1}(y)$. It had also been shown in the proof of proposition 5 that the total of fire sales k resulting from strategy z, are: $k = h(z) = \frac{z^{(\theta+1)}}{\theta+1}$. Again, this function is monotonously increasing and can be inverted $z = h^{-1}(k)$: $z = h^{-1}(k) = \left((\theta+1)k\right)^{\frac{1}{\theta+1}}$. Inserting this into y=f(z) generates the monotonous and invertible relationship between liquidity provision and fire sales:

$$y = g^{-1}(k) = \frac{\delta}{\delta+1} + \frac{\left(\left((\theta+1)k\right)^{\frac{1}{\theta+1}}\right)^{\delta+1}}{\delta+1} - k \tag{8}$$

The rest of the statements made in proposition 6 can be shown easily by inserting the relevant parameter values in the functional forms derived.□

The following figure shows the liquidity possibility sets under the two strategies (i.e. strategy to fire sale the most liquid assets, and pledging the rest, and the reverse order) for (δ, θ) = (0.5, 2), (0.1, 10). The horizontal axis contains the fire sale costs k, while the vertical-axis maps the liquidity provision. This also illustrates proposition 4 in the sense that indeed the reverse strategy is dominated.

Figure 7: liquidity generation / fire sales trade-offs for the efficient and the reverse strategy and two alternative parameter combinations

(δ, θ)	
(0.5, 2)	(0.1, 5)
	

6. Stable funding structure with the lowest possible cost

In the previous sections, it was assumed that the bank balance sheet was given, and the question was under what circumstances the bank can achieve stability of short term funding, i.e. the absence of a self-fulfilling run problem. It was shown that depending on its liability mix, the haircut (δ) and asset liquidity parameters (θ) the bank was able or not to achieve this. This section will make the liability structure endogenous in a very simple setting. In a full equilibrium model, one would also need to model household preferences and the decisions on investment projects. For the purposes of this paper, one can simplify and assume instead that different liabilities require different remuneration rates but are at these rates perfectly elastic.[1] In this setting, total bank funding cost will be a proxy for the ability of banks to deliver on maturity transformation, which is an essential contribution of banking to society.

For given, deterministic δ, θ, competing banks will always **go to the limit in terms of the cheapest possible liability structure** as determined by the conditions in the strategic game, such that the no-run equilibrium is still maintained as SNNR equilibrium. Assume that the cost of remuneration of the three asset types are r_e for equity, r_t for term funding, and 0 for short term deposits. Also assume that $r_e > r_t > 0$, and that $\theta > \delta$. What will in this setting be the composition of the banks' liabilities? The objective of choosing a liability composition will be to minimize the average overall remuneration rate subject to maintaining a stable short term funding basis. One strategy could be to aim at $\delta/(\delta+1) \geq (1-e-t)/2$, such that fire sales will not be needed at all as backstop. If fire sales are not needed, then term funding is superior to equity and equity will be set to zero, i.e. liabilities will consist only in term funding t and in the two short term deposits (1-t)/2. Therefore the condition for stable short term funding will be $\delta/(\delta+1) \geq (1-t)/2 \Rightarrow t^* = 1-2\delta/(\delta+1)$. The average remuneration rate of bank funding would be $t^* r_t$. A second strategy would be to rely only on the fire sales approach but to hold the necessary equity. This would mean that the two minimum conditions to be fulfilled are $\theta/(1+\theta) = (1-t-e)/2$ and $e = ((1-t-e)/2)^{(1+\theta)}/(1+\theta)$. These conditions can be solved for a unique optimum t* and e*, and hence for the average necessary remuneration rate of bank liabilities $t^* r_t + e^* r_e$.

The **general problem of optimal liquidity management** is to minimise through the choice of t, e and z (each in [0,1]), the average remuneration rate of the banks' liabilities $t^* r_t + e^* r_e$, subject to the conditions $\frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1} \geq (1-t-e)/2$ and $\frac{z^{(\theta+1)}}{\theta+1} \leq e$.

Below some illustrative results of this optimisation problem are presented for different values of the liquidity and haircut parameter. **Table 1 varies the collateral framework** of the central bank as captured through the parameter δ. All other exogenous variables are kept constant. The table reveals that in this example, the *less* restrictive the central bank collateral framework,

- The higher the equilibrium share of short term funding

[1] Although this is not done here, one could also apply the model to endogenize the liquidity parameters of banks' assets. One could imagine that the bank has a "production function" to improve asset liquidity, and that this function can be expressed as the cost to increase δ, θ. The investment into improved asset transparency / liquidity would be v, and one could assume two functions $\delta(v)$ and $\theta(v)$ with $\delta(0)= \delta0$ and $\theta(0)= \theta0$ and $d\delta/dv > 0$ and $d\delta/dv < 0$, as well as $d\theta/dv > 0$ and $d\theta/dv < 0$. Ways to improve the liquidity of assets are for instance to (i) standardise assets (e.g. by standardising claims at origination); (ii) originate claims only to standard and transparent projects (foregoing the higher return properties of idiosyncratic, very information intense projects) (ii) securitise assets; (iii) develop information systems that capture asset characteristics and risk factors; (iv) allow for third party asset review exercises; etc. This idea, that would be an ingredient of a full equilibrium model, is not taken further here, and hence the focus in the rest of the paper is only on the cheapest stable liability structure.

- The lower the equilibrium share of term funding
- The lower the equilibrium share of equity
- The lower the equilibrium ratio between equity and term funding
- The lower the potential role of asset fire sales relative to central bank credit
- The lower the funding costs of banks and hence the lower, in competitive equilibrium, the costs of bank funding to the real economy.

Table 1: Impact of central bank collateral framework on banks' liability structure and cost

Exogenous parameters					
δ	**0.01**	**0.1**	**0.2**	**0.5**	**1**
Θ	1				
r_t	2%				
r_e	10%				
Results					
t	0.39	0.39	0.38	0.29	0.00
e	0.05	0.04	0.03	0.01	0.00
Implied short term funding (1-t-e)	0.56	0.57	0.59	0.70	1.00
Share of assets foreseen for fire sales (z)	0.33	0.29	0.25	0.11	0.00
Refinancing costs of bank	1.32%	1.21%	1.08%	0.64%	0.00%

Normally, one would assume that the central bank collateral framework is designed on the basis of considerations outside the present model (in particular risk protection). However, the results above illustrates that whatever the reasons for the design of the framework, the framework will matter for (i) the equilibrium liability structure of banks, (ii) the role of central bank funding in case of a bank run; (iii) the likelihood of negative asset fire sales externalities in case of a bank run; (iv) the stance of monetary policy; (v) the efficiency of bank intermediation as measured by funding costs of the real economy. In sum, the central bank collateral framework seems to be of pervasive importance not only for the banking system, but also for monetary policy and optimal banking regulation. One may also note that a sudden non-anticipated tightening of the collateral framework can cause a bank run as it may change the strategic away from the SNNR equilibrium. The triggering of bank runs by central banks who suddenly limit the borrowing potential of banks is illustrated for example by Bagehot (1873) and King, (1936) for the 19th century Bank of England, or by Priester (1932), for the Reichsbank decision of 13 July 1931 (see also Bindseil and Winkler, 2012).

Table 2 varies the parameter capturing asset liquidity. Asset liquidity is likely to vary over time with the liquidity cycle, and will deteriorate in particular in a liquidity crisis. Moreover, the liquidity of assets may change structurally over time with changes of market infrastructure and IT systems, asset standardisation, new securitisation techniques etc. In this example, the higher θ, i.e. the higher the asset liquidity:

- The higher the equilibrium share of short term funding
- The lower the equilibrium share of short term funding
- Interestingly, the equilibrium share of equity first increases, and then decreases again
- Also the share of assets foreseen for fire sales first increases and then decreases again. The increase may be seen positive if one would like to see independence from the central bank. However, it could also create problems if it is tested in combination with a crisis related deterioration of asset liquidity, as it may lead to particular asset fire sales dynamics.

- The lower the funding costs of banks and hence the lower, in competitive equilibrium, the costs of bank funding to the real economy. This can be understood as a pro-cyclical element. In a downturn, when credit risk and information asymmetries increase, asset liquidity will decrease and will thus lead to an increase of bank equilibrium funding costs, even if the concrete occurrence of a bank run can be avoided as the bank could adjust its funding structure sufficiently early.

Table 2: Impact of asset liquidity on banks' liability structure and cost

Exogenous parameters						
δ	0.1					
θ	**0.4**	**0.7**	**1**	**1.5**	**2**	**4**
r_t	2%					
r_e	10%					
Results						
t	0.79	0.60	0.39	0.11	0.00	0.00
e	0.00	0.03	0.04	0.06	0.04	0.01
Implied short term funding (1-t-e)	0.21	0.37	0.57	0.84	0.96	0.99
Share of assets foreseen for fire sales (z)	0.03	0.16	0.30	0.46	0.51	0.49
Refinancing costs of bank	1.62%	1.47%	1.21%	0.78%	0.43%	0.05%

The non-monotonous behaviour of the equilibrium equity share and the share of assets foreseen for fire sales shows how complex, even in a simple model, the relationship between the various variables can be, and hence how careful one needs to be in interpreting developments and designing a regulatory framework. The table also confirms how dangerous non-anticipated declines in asset liquidity can be as they reduce the sustainable size of short term funding. If supervisors have insights into this problem, while banks have not, then regulation can be an adequate response.

The following tables vary both asset characteristic parameters and show the impact on three key endogenous variables: sustainable share of short term funding, funding cost, and share of asset foreseen for fire sales (starting from the most liquid assets).

A number of additional observations can be made on the basis of table 3. Part (a) of the table for instance shows that with very favorable asset liquidity or a very favorable collateral framework, banks will tend to fund their entire balance sheet with short term liabilities. Part (b) of the table shows for instance that the effectiveness of a broadening of the collateral framework (an increase of δ) in terms of reducing funding costs of banks (which might be relevant at the zero lower bound) is particularly relevant for banks with assets of low liquidity (with a low θ). Part (c) of the table reveals that in case of a very favorable collateral framework, the role of central bank reliance tends to 100% (and the role of fire sales to zero), while the role of asset fire sales never exceeds 52%, even if the collateral framework is very tight and the fire sales discounts are low.

Table 3: Impact of the two asset liquidity parameters on key endogenous variables

(a) Sustainable share of short term funding

		δ				
		0.01	0.10	0.20	0.50	1.00
θ	0.40	16%	21%	34%	67%	100%
	0.70	34%	37%	42%	67%	100%
	1.00	56%	57%	59%	69%	100%
	2.00	95%	96%	96%	91%	100%
	4.00	99%	99%	100%	99%	100%

(b) Average funding costs

		δ				
		0.01	0.10	0.20	0.50	1.00
θ	0.40	1.88%	1.62%	1.30%	0.66%	0.00%
	0.70	1.64%	1.47%	1.26%	0.66%	0.00%
	1.00	1.32%	1.21%	1.08%	0.65%	0.00%
	2.00	0.50%	0.43%	0.40%	0.29%	0.00%
	4.00	0.06%	0.05%	0.05%	0.05%	0.00%

(c) Part z of assets foreseen for fire sales (starting from most liquid)

		δ				
		0.01	0.10	0.20	0.50	1.00
θ	0.40	11%	3%	0%	0%	0%
	0.70	20%	16%	12%	0%	0%
	1.00	33%	29%	25%	7%	0%
	2.00	52%	51%	49%	35%	0%
	4.00	50%	49%	47%	43%	0%

7. The impact of regulation on the liability structure and on funding cost

Now the model and bank optimization problem developed in the previous section is used to understand the effects of minimum capital and an LCR type liquidity regulation. Afterwards, it is discussed how to rationalize regulation in the light of the model. Table 4 shows the impact of various levels of minimum capital requirements (simple minimum levels of e) starting again from the standard parameter set. It may be noted that without capital requirements, the equilibrium equity level is 0.04, and hence only capital requirements above 0.04 are binding. In this example, the higher the capital requirement:

- The lower the equilibrium share of long term funding
- The equilibrium share of short term funding increases when equity levels are pushed higher due to capital adequacy requirements, however of course it starts to fall again once long term funding has been completely crowded out and hence the increase of equity must go at the expense of the share of short term funding.
- The role of asset fire sales relative to the role of central bank lending (z) increases. This may be considered positive if one would like to see independence from the central bank, but it may be

seen as negative if one considers that the role of asset fire sales can be pushed to excessive levels such that it includes less liquid assets for which the fire sale externality can be material.

Table 4: Impact of minimum capital requirements on banks' liability structure and cost

Exogenous parameters					
δ	**0.1**				
θ	1				
r_t	2%				
r_e	10%				
Minimum capital requirements	**0.00**	**0.05**	**0.10**	**0.20**	**0.30**
Optimization parameters					
t	0.39	0.36	0.17	0.00	0.00
e	0.04	0.05	0.10	0.20	0.30
Implied short term funding (1-t-e)	0.57	0.59	0.73	0.80	0.70
Share of assets foreseen for fire sales (z)	0.29	0.32	0.45	0.52	0.52
Refinancing costs of bank	1.21%	1.21%	1.34%	2.00%	3.00%

To interpret the impact of minimum liquidity requirements in the form of the LCR, the concept of HQLA needs to be specified within the model. One can for instance define that an HQLA is an asset for which fire sale losses are not higher that 10%. For the chosen case θ=1 this means that the first 10% of assets are HQLAs. Therefore, an LCR requirement of 0.25 means that short term funding can be up to 0.4, an LCR requirement of 1 means short term funding of a maximum of 0.1 etc. In fact, in the assumed case with a given asset composition, the LCR becomes equivalent to a regulation that sets a maximum share of short term liabilities.

Table 5: Impact of LCR requirement on bank liability structure and cost

Exogenous parameters					
δ	**0,1**				
θ	1				
r_t	2%				
r_e	10%				
Minimum LCR	**-**	**0.25**	**0.50**	**1**	**2**
Implied maximum of short term funding	**1**	**0.40**	**0.20**	**0.10**	**0.06**
Optimization parameters					
t	0.39	0.59	0.80	0.90	0.95
e	0.04	0.01	0.00	0.00	0.00
Implied short term funding (1-t-e)	0.57	0.40	0.20	0.10	0.05
Share of assets foreseen for fire sales (z)	0.29	0.16	0.02	0.00	0.00
Refinancing costs of bank	1.21%	1.30%	1.60%	1.80%	1.90%

The liquidity regulation is binding already with an LCR requirement of 0.25. Tightening the LCR requirement (i.e. increasing the minimum LCR = reducing the maximum share of short term assets) has the following effects:

- It increases the equilibrium share of long term funding
- It reduces the equilibrium level of equity (absent binding capital adequacy regulation)
- It reduces the equilibrium share of short term funding

- It decreases the role of fire sales, relative to the role of central bank pledging of assets. This might appear counterintuitive as the intention of the regulation is also to reduce reliance on the central bank in case of a liquidity crisis
- It increases refinancing costs of banks (and hence, in a competitive equilibrium of the banking system, the refinancing costs of the real economy)

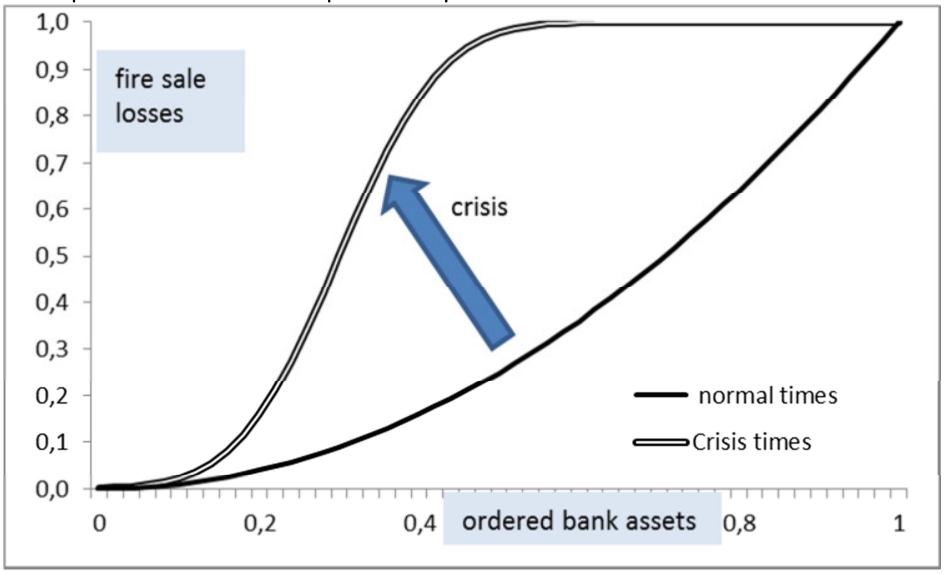
The counterintuitive result that liquidity regulation strengthens the role of potential reliance on central bank funding relative to the potential role of fire sale can be interpreted as follows: since liquidity regulation for a given set of assets leads only to an increased share of long term borrowing, the total need for liquidity buffers decreases relative to the available buffers, and therefore to save on the most expensive type of liabilities, namely equity, the bank can rely more and more on central bank credit alone as source of liquidity.

To revisit **more generally the rationale of liquidity regulation** requires obviously going beyond the equilibrium impact as described above. The impact on the equilibrium liability structure and on bank funding costs in itself is relevant in assessing the overall merits of regulation, but is not sufficient to justify regulation. What the equilibrium model presented above does not really capture is how the liquidity buffers foreseen to sustain short term funding are actually tested. In equilibrium, they would according to the model never be tested, and hence in fact no liquidity regulation would be needed, allowing for the lowest cost of bank intermediation for a given θ, δ.

One approach to justify regulation would be some ad hoc assumption on the existence of exogenous *non-anticipated shocks* that will lead to an actual testing of buffers. Assume for instance that once in a while, asset liquidity as captured by θ suddenly declines, and neither banks nor the short term creditors can anticipate this for some reason. If the shock to θ is sufficiently large, it can change the strategic game between the two short term bank creditors in a way to lead to a bank run, leading to socially undesirable fire sale losses. *Alternatively*, and maybe more convincingly, one may assume that banks anticipate shocks to the liquidity parameters including the fact that these shocks can lead to bank runs and trigger costly asset fire sales. This could be taken into account by banks who would therefore foresee buffers. For instance, if the maximum stable short term funding that can be sustained for a given combination of θ, δ is $s^*(\theta, \delta)$, then banks would instead choose $s^\# < s^*$, whereby the exact $s^\#$ would be based on an optimization calculus in view of the probability distribution assigned to future values of θ, δ. However, the problem would still be that banks would not factor in the negative externalities of fire sales, or the dislike of public authorities on large recourse to central banks. Through liquidity regulation, an even lower $s^\Diamond$ could be imposed that would reduce the probability of bank runs, fire sales and large central bank reliance accordingly. Example: Assume that normally θ=1 and δ=0.1, and that r_e =10% and r_t =2% and r_s =0. As shown above, the representative bank in competitive equilibrium will then have t=0.39, e=0.04, s=0.57 z=0.29 r= 1.21%. Now assume that, in any period, with probability of 1%, θ declines suddenly for one period to 0.4, implying that the sustainable short term funding is only 0.21. Assume that indeed in these case a bank run starts leading to bank default and liquidation, and hence to costs equal to 1/(1+0.4)=0.71. Knowing this probability and the consequences, the bank could decide to always choose a liability structure such as to sustain stable short term funding even for the periods in which θ declines to 0.4. As shown, this leads to refinancing costs of 1,62%, i.e. 0.41% higher. The risk neutral bank will thus compare the 0.41% total higher funding costs with the 71% asset value destruction which occurs with probability of 1%, and will chose the more expensive liability structure that always sustains stable short term funding, i.e. the social optimum will prevail anyway, without banking regulation. If however the sudden deterioration of asset fire sales liquidity to 0.4 occurs only with probability of 0.5%, the calculus of the representative bank in competitive equilibrium leads it to choose the cheaper funding structure and to accept that once in a while bank runs will occur with bank failures. If there are additional social cost (negative externalities) of bank failures, the decisions taken by the representative bank will no longer necessary be socially optimal. If for example negative externalities of default occur which are as big as the direct damage to the bank assets, then, from a social perspective it would be desirable that the bank chooses the more solid liquidity structure. In this case there is a rationale for liquidity regulation.[2] Of course, this does not prove the optimality of liquidity regulation as defined by Basel Committee (2013). Regulation can take a different form, as illustrated by e.g. Perotti and Suarez (2011).

Beyond this very simple example, a number of model sophistications should be considered to allow for a more qualified analysis of regulation. For example, one could assume: (i) continuous probability functions for future values of the liquidity parameters θ and δ; (ii) that each depositor receives every period a noisy independent signal on θ, which leads sometimes to the run by only one depositor, sometimes to a run by both, and mostly to a run of none; (iii) that also the central bank collateral framework can change over time, maybe with δ and θ being correlated (in fact pro-cyclical elements of collateral frameworks, such as marking-to-market or rating dependence of eligibility and haircuts, will lead to a decline of δ in a crisis); (iv) that in a crisis, the change of asset liquidity can no longer be described well in the framework as an exponential function and its exponent θ, but that typically HQLAs do not deteriorate in their asset liquidity, while non-HQLAs do. The following figure captures this idea.

Figure 8: shift of the asset liquidity curve in a crisis that cannot be captured by a simple decrease of the exponential parameter θ



Ideally, a model would capture different degrees of liquidity stress materializing (with respective probabilities), and in particular (i) partial runs that are compensated by fire sales and central bank recourse and (ii) runs exceeding the buffers and leading to default due to illiquidity or insolvency. To assess deviations from the social optimum that occur without regulation, and to calibrate optimal regulation, one obviously needs a clear view on the extent of fire sales externalities (for different

[2] Note that in this example, the losses that occur in case of default need to be allocated to the different liability holders according to the seniority of claims, and the different liability holders would require ex ante a credit risk premium relative to the basic interest rates as assumed, such that, assuming risk neutrality, all creditors of the bank are duly compensated. This does not change the eventual economic problem and the issue of externality that can justify liquidity regulation.

degrees of asset liquidity), the social costs of large temporary reliance on the central bank, and the negative externality of bank default. Then, eventually, conclusions could be drawn on the overall welfare properties of banking regulation, encompassing both effects of regulation, namely (i) the higher funding costs of banks in normal times, and (ii) the lower likelihood and costs of bank runs. It would obviously have to be assumed that in case of a non-anticipated deterioration of asset liquidity leading potentially to a bank run, the regulatory ratios would be allowed to be breached temporarily, a key issue that requires further elaboration as also acknowledged in Basle Committee (2013).

8. The central bank collateral framework as a policy tool?

During the financial crisis, central banks have taken a wide range of collateral measures to increase the potential recourse to central bank credit (see e.g. Markets Committee, 2009, ECB, 2013). While some measures were launched as soon as 2007 and intensified in 2008, others were taken only recently and could be interpreted as relating to the reaching of the zero lower bound, such as the Funding for Lending scheme of the Bank of England (Chrum and Radia, 2012).

As the **essential purpose of the central bank collateral framework is risk protection**, the observed collateral policy measures raise the question what exactly the intentions of the central banks have been to widen collateral availability (and hence potential central bank recourse) in particular in a context of deteriorating asset liquidity. The standard explanation provided by central banks and taken up in the critical perception of regulators as referred to in section 1 has been that this increase allowed the banks to substitute for the impossibility to roll over short term funding. The model proposed in this paper allows for **new interpretations of collateral widening as a policy measures**:

- First, when θ declines suddenly, increasing δ is a way to preserve the no-run equilibrium (the SNNR) and thereby is a necessary condition to *prevent* increases in central bank reliance, fire sales, and/or defaults. In this sense, it benefits all banks and financial stability in general, and not only those banks who already experience an actual run. Moreover, it may be noted that the model provides support to Bagehot's (1873) "inertia principle" according to which the central bank should not tighten its collateral framework in a financial crisis as a reaction to the deterioration of asset liquidity: "If it is known that the Bank of England is freely advancing on what in ordinary times is reckoned a good security—on what is then commonly pledged and easily convertible—the alarm of the solvent merchants and bankers will be stayed…" Lowering δ when anyway θ declines would mean to decrease particularly strongly the amount of sustainable short term funding and thereby to maximize the probability of a destabilization of bank funding, contributing, instead of preventing, large central bank recourse and fire sales of assets.
- Second, assuming that a deterioration of θ can be anticipated as a crisis is building up, one could imagine in principle *that banks can adjust their liability structure in time*. If this would indeed be possible, it is however likely that it would come at very high costs because in such a context also investors will have a strong preference for short term assets and the collective attempt of all banks to increase the maturity of their liabilities will therefore lead to an enormous increase of bank funding costs. This would be rather pro-cyclical, and an adjustment of the collateral framework parameter δ could be seen as a policy tool to prevent such a steep increase of funding costs.
- While the effect described in the previous bullet point could at least in theory also be addressed by conventional monetary policy, i.e. a lowering of central bank credit operations rates, this has limits as far as the zero lower bound is reached. When this limit is reached, then a widening of collateral availability may become relevant as an alternative approach to lowering effective bank funding and hence lowering the cost of funding of the economy (in the same way as large scale asset purchase programs contribute, when the zero lower bound has been reached, to lower the costs of long term funding to the economy, in particular in a capital markets based financial system).

An interesting question is how to **reconcile such policy considerations** which seem to suggest an increase of δ **with the original goal of the collateral framework** and its specification through δ as a **risk management tool**. Three interpretations appear to be possible.

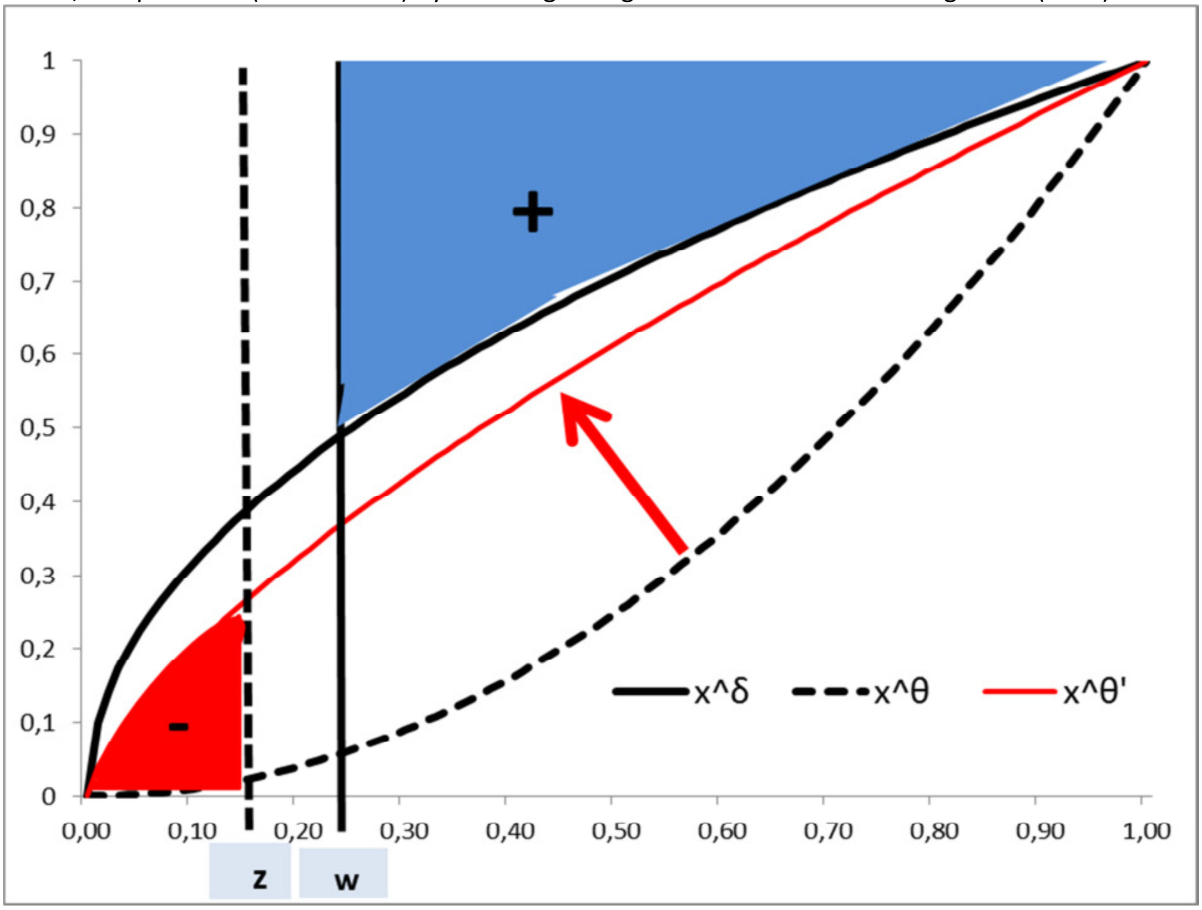
- First, it has been argued that in a liquidity crisis, risk parameters are endogenous to central bank action, and therefore a broadening of the collateral availability through a lowering of haircuts could under some circumstances decreases the risk taking of the central bank and reduce its eventual losses (and vice versa, a tightening of the collateral framework could increase eventual central bank losses). As Bagehot (1873) formulated it, "only the brave plan" would be the "save plan" for the Bank of England (see Bindseil and Jablecki, 2013, for a model of this conjecture).
- Second, it could be argued that the central bank has multiple objectives, and that the collateral framework impacts on several of its objectives, while the central bank does at least under some circumstances not have enough tools to achieve all goals perfectly. In such a case, minimizing some convex loss function with regard to missing the various goals may lead the central bank to accept not achieving its normal risk control objectives through the collateral framework. For example, if a central bank has calibrated its collateral framework in a way that it is protected in case of a counterparty default with 99% against incurring a loss when liquidating the collateral portfolio, then it may accept under extraordinary circumstances, and taking into account the other effects of the collateral framework in achieving its goals (such as first of all price stability, but also financial stability and social welfare in general), to lower this confidence level to 95%.
- Third, it could be argued that in fact central banks can increase collateral availability in a crisis without changing the parameter δ and therefore without changing risk taking through the following approach: pre-crisis, they can declare a part of the less liquid collateral as completely non-eligible (i.e. apply a 100% haircut, above the haircut that would have been applied by δ), but remove this partial "collateral deactivation" without changing δ when a liquidity crisis materializes. This approach, which seems to describe fairly well what many central banks have been doing during the financial crisis, is developed further in the next section.

A central bank strategy to keep ineligible in normal times less liquid assets, but to consider making them eligible (without changing δ, i.e. while preserving risk equivalence) in crisis times

The central may want to pursue a strategy to make in normal times less liquid assets ineligible, which could be seen to have the following advantages. First, assessing and transferring these assets to the central bank is costly both for the counterparty and for the central bank in view of the nature of these assets. Second, in view of the high haircuts, this administrative cost per unit of collateral is even higher in relation to the central bank funding potentially provided. Third, keeping assets ineligible in normal times, but making them potentially eligible in times of non-anticipated crisis, may allow to preserve financial stability without changing the parameter δ of the collateral framework, i.e. without changing the basic risk control parameter of the central bank, which is good from the risk management perspective.

Suppose that a central bank considers that as a matter of principle, it will not accept assets as eligible in normal times for which haircuts deserve to be higher than 50%. Call w the asset share at which the central bank cuts off eligibility. Then w is easily obtained as $w^{\delta} = 0.5 <=> w = 0.5^{\left(\frac{1}{\delta}\right)}$. For example, if δ=0.5, then w=25%. In this case, the maximum liquidity that can be obtained from the central bank is $0.25 - \frac{0.25^{\delta+1}}{1+\delta} = 0.1667$. The collateral buffer that the central bank could activate by accepting in a crisis all assets would be $\frac{\delta}{1+\delta} - 0.1667 = 0.1667$. In other words, the central bank could double in this case central bank refinancing power from 0.1667 to 0.333 without giving up risk equivalence. When modelling the ex ante liquidity risk management strategy of the bank, and setting an optimal strategy encompassing fire sales and central bank reliance, then, obviously one would need to assume that the counterparty would not anticipate the relaxation (otherwise, in the model it would be as if from the start the entire assets would be eligible). The ad hoc assumptions that the banks do not anticipate the relaxation seems to be fair in view of the rareness of liquidity crisis and the constructive ambiguity that central banks apply. It is possible to calculate through the optimisation problem to determine z (the split of asset fire sales and central bank recourse in the liquidity strategy of the bank) under the central banks' strategy to declare ineligible all assets for which haircuts would be higher than 50%. Then, it is possible to calculate the critical value $\theta^{\#}$ relative to the initial θ that can still be accommodated by the central bank making all assets eligible and hence adding an extra liquidity buffer of 0.1667 to the bank. The calculus is omitted here, but the idea is illustrated in figure 9. Suppose for instance that with θ=2, δ=0.5, a maximum haircut for eligible assets of 0.5 and hence w=25%, the optimal liquidity management strategy (for some assumed costs of the different liabilities) would be characterised by z = 0.15. Then, in case that due to a liquidity crisis, asset liquidity deteriorates such that θ'=0.7 implies a change of collateral liquidity buffer of $\frac{0.15^{2+1}}{2+1} - \left(\frac{0.15^{0.7+1}}{0.7+1}\right) = -0.0223$. This is below the buffer that the central bank can restore by making all assets eligible, and hence the central bank can prevent a destabilisation of deposits without putting into question her risk control parameter δ=0.5

Figure 9: Loss of liquidity buffers (red surface) reflecting deterioration of asset liquidity parameter θ from 2 to 0.7, compensated (blue surface) by widening of eligible asset set without change of δ (= 0.5).



9. Conclusion

The aim of this paper was to deepen the understanding of the interaction between bank asset liquidity, the central bank collateral framework, and liquidity regulation, and to better understand the effects of these factors for financial stability and the ability of the banking system to deliver maturity transformation, which is one of its key functions for society. It was assumed that liquidity and collateral treatment of assets can be approximated by exponential functions, and that banks in a competitive equilibrium choose the cheapest possible liability structure that ensures stability of short term funding. Stability of short term funding was modeled as strategic game of two short term depositors who had the options to keep their deposits with the bank or to "run". After establishing the conditions for stability of short term funding, the effects of the key exogeneous variables (asset liquidity, central bank collateral framework, regulation) on the endogenous variables were derived, namely on (i) the liability structure (the mix of short term funding, long term funding, and equity), (ii) the reliance of the two emergency funding sources (fire sales, central bank credit), and on (iii) the funding costs of banks, as proxy of the banks' ability to deliver maturity transformation. The following table summarizes the main effects (starting from a base case with a certain parameter specification).

Table 6: Effects of changes of exogeneous variables on endogenous variables (first derivative)

Endogenous variables → **Exogeneous variables ↓**	Share of equity in total liabilities **e**	Share of long term debt in total liabilities **t**	Share of short term debt in total liabilities **1-t-e**	Share of bank assets foreseen for fire sales (starting from most liquid assets) vs. central bank reliance **z**	Refinancing costs of banks $\mathbf{er_e + tr_t}$
CB Collateral δ	**-**	**-**	**+**	**-**	**-**
Fire sales costs θ	**+/-**	**-**	**+**	**+/-**	**-**
Required LCR	**-**	**+**	**-**	**-**	**+**
Requ. capital e	**+**	**-**	**+/-**	**+**	**+**

Remark: "+/-" means that the first derivative is first positive and then beyond some point negative

The identification of these equilibrium effects allows to better understand and to revisit the likely effectiveness of liquidity regulation with regard to achieving certain goals, such as e.g. "reduce reliance on central banks as source of emergency liquidity", "avoid that in a crisis banks fire sale non-liquid assets", etc. The equilibrium effects developed in this paper seem to be a necessary first step for a full understanding of the effects of liquidity regulation.

For a comprehensive revisiting of the rationale and effects of liquidity regulation, the model needs to be expanded to capture more explicitly dis-equilibrium states in which the liquidity buffers are actually used by banks (leading to fire sales, large central bank reliance, or bank defaults, all associated with some social cost that needs to be well understood). In this paper, only some general principles of how liquidity regulation can enhance financial stability were developed with disequilibrium states, and some illustrative examples were provided. Accordingly, liquidity regulation imposes excess liquidity buffers in normal times beyond those that banks would hold voluntarily, such that sudden declines of asset liquidity trigger with less probability a destabilization of short term liabilities and the associated negative externalities.

It was also shown how changes of the collateral framework can be a financial stability policy tool, matching the observation that most central banks in the current financial crisis tended to extend collateral eligibility. Finally, it was noted that the identified impact of asset liquidity and of the central bank collateral framework on funding costs of banks is relevant for monetary policy for at least two reasons: first, policy makers need to be aware that a tightening of any of the two emergency liquidity sources needs to be, everything else unchanged, compensated by a lowering of the monetary policy interest rate to maintain unchanged funding costs of the real economy. Second, when the central bank has reached the zero lower bound, and therefore cannot use standard interest rate policies any longer to lower the money rate, it can consider to use its collateral framework to reduce actual funding costs. The use of the collateral framework for policy purposes (financial stability and monetary policy) has of course to take place with due consideration to the original purpose of the collateral framework, which is the protection of the central bank

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Annex 1: Proof of proposition 2

Proposition 2: If $\delta = 0$ an SNNR equilibrium exists if and only if (1-t-e)/2 ≤ θ/(θ+1) and e ≥ ((1-t-e)/2)$^{\theta+1}$/(θ+1)

To prove proposition 2, the cases as shown in figure A1-1 are examined sequentially.

Figure A1-1: Existence of a unique no-run equilibrium case when $\theta \in [0,1], \delta = 0,\ e \in [0,1], \varepsilon > 0$

		Liquidity sufficient for the withdrawal of		
		(A) Both depositors: (1-t-e)<θ/(θ+1)	(B) Only one depositor: (1-t-e)/2 < θ/(θ+1) < (1-t-e)	(C) Not even one depositor θ/(θ+1) < (1-t-e)/2
Capital sufficient for the fire sales losses associated with the withdrawal by	(i) At least one depositor: e >= ((1-t-e)/2)$^{\theta+1}$/(θ+1)	***Unique stable no-run equilibrium***		***No unique stable no-run equilibrium***
	(ii) Not even one depositor: e >= ((1-t-e)/2)θ+1/(θ+1)	***No unique stable no-run equilibrium***		

The proposition can be verified by again establishing the strategic game pay-offs and verifying conditions $U_1(K_1, K_2) > U_1(R_1, K_2)$ and $U_1(K_1, R_2) > U_1(R_1, R_2)$. Consider the following strategic game representations. Figure A1-2 represents the strategic game for the cases (Ai). Indeed, in this case, depositors never suffer losses and therefore the conditions for the SNNR are verified.

Figure A1-2: The bank run game in case (Ai)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε (1-t-e)/2
	Run	(1-t-e)/2 (1-t-e)/2-ε	(1-t-e)/2-ε (1-t-e)/2-ε

In the case (Aii) depositors face no loss when they both run, but depositor 1 faces a loss if only depositor 2 runs because of the insolvency of the bank and the pari passu ranking of all remaining depositors (short term and long term). Call RR the recovery ratio which is strictly below 1 in this case. The total asset values remaining to satisfy the other short term depositor (who have claims of (1-t-e)/2) and the long term depositors (who have claims of t) will be the liquidation value of assets minus what needed to be already paid out to the withdrawing depositor: θ/(θ+1) - (1-t-e)/2. The recovery ratio RR will therefore be:

$$RR = \frac{\frac{\theta}{\theta+1} - \frac{1-t-e}{2}}{\frac{1-t-e}{2} + t} = \frac{\frac{2\theta}{\theta+1} - 1 + t + e}{1 + t - e} \qquad \text{(A1-1)}$$

It is easily verified that RR<1. The strategic game in case (Aii) is as depicted in the following figure.

Figure A1-3: The bank run game in case (Aii)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε RR(1-t-e)/2
	Run	RR(1-t-e)/2 (1-t-e)/2-ε	(1-t-e)/2-ε (1-t-e)/2-ε

In this case, the condition $U_1(K_1, R_2) > U_1(R_1, R_2)$ is not satisfied since $U_1(K_1, R_2) =$ RR(1-t-e)/2 and $U_1(R_1, R_2)$ = (1-t-e)/2-ε and RR(1-t-e)/2 < (1-t-e)/2-ε as ε is assumed to be a small number. Therefore, the no-run strategies is not a dominating strategy.

Now consider (Bi), i.e. (1-t-e)/2 < θ/(θ+1) < (1-t-e) and e ≥ $((1\text{-}t\text{-}e)/2)^{\theta+1}/(\theta+1)$. In case only one depositor runs, default is avoided both from a liquidity and from the solvency perspective. If both depositor runs at once, then they will each recover (θ/(θ+1))/2, but then nothing is left of the company in terms of assets and hence for the remaining deposits the recovery ratio is zero. An SNNR equilibrium applies since the conditions $U_1(K_1, K_2) > U_1(R_1, K_2)$ and $U_1(K_1, R_2) > U_1(R_1, R_2)$ are both fulfilled.

Figure A1-4: The bank run game in case (Bi)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε (1-t-e)/2
	Run	(1-t-e)/2 (1-t-e)/2-ε	(θ/(θ+1))/2 (θ/(θ+1))/2

Now consider (Bii), i.e. (1-t-e)/2 < θ/(θ+1) < (1-t-e) and e < $(((1\text{-}t\text{-}e)/2)^{\theta+1}/(\theta+1)$. In this case, if one depositor runs, this is enough to trigger solvency induced default, and hence the second depositor will face losses.

Figure A1-5: The bank run game in case (Biii)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε RR(1-t-e)/2
	Run	RR(1-t-e)/2 (1-t-e)/2-ε	(θ/(θ+1))/2 (θ/(θ+1))/2

It is easy to show that RR(1-t-e)/2 < (θ/(θ+1))/2. It follows from the fact that the available total asset liquidation value θ/(θ+1) is identical across all default scenarios, and that from the perspective of depositor 1, if only depositor 2 runs not only depositor 2 can recover more than if both run, but also the term depositors will still get their part of the share (they get nothing if both runs). Therefore, again, the condition $U_1(K_1, R_2) > U_1(R_1, R_2)$ is not satisfied. Therefore, there is no SNNR equilibrium.

Now consider the case (C) in which not even one depositor can withdraw without triggering liquidity induced default, i.e. θ/(θ+1) < (1-t-e)/2. In this case, depositors will face losses whenever one depositor runs. The question is whether the pay-off is better if both run, relative to the case that only the other depositor runs. This is the case since again the total asset liquidation value is the same across all cases, and in case the other depositor runs he is able to withdraw more value for himself, and hence leaves less value for all others. In fact in case (C), the remaining value for others is zero (RR=0) as all assets are sold to satisfy as much as possible the short term depositor(s) who withdraw. Therefore, in all cases of C, the condition $U_1(K_1, R_2) > U_1(R_1, R_2)$ is not fulfilled.

Figure A1-6: The bank run game in case (Ci), (Cii)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(θ/(θ+1)) 0
	Run	0 (θ/(θ+1))	(θ/(θ+1))/2 (θ/(θ+1))/2

This completes the verification of proposition 2.

Annex 2: Proof of proposition 4

Proposition 4 states that in funding strategies to address withdrawals of short term deposits relying on both funding sources, the bank should always foresee to fire sale the most liquid assets and pledge the rest. It is shown in the proof of proposition 6 that when applying the strategy to foresee fire-selling the share z of most liquid assets, and to pledge the part (1-z) of the least liquid assets, one obtains as possibility set the following monotonous and invertible relationship between liquidity provision and fire sales:

$$y = \frac{\delta}{\delta+1} + \frac{((\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1} - k$$

(A2-1)

One should first derive a similar possibility set for the reverse strategy. The central bank liquidity from fire selling assets (1-z) starting from the least liquid is:

$$y_1 = (1-z) - \int_z^1 x^\theta \, dx = (1-z) - \left(\frac{1^{\theta+1}}{\theta+1} - \frac{z^{\theta+1}}{\theta+1}\right) = 1 - z + \frac{z^{\theta+1}-1}{\theta+1}$$

(A2-2)

The liquidity generated from pledging assets z starting from the most liquid is:

$$y_2 = z - \int_0^z x^\delta \, dx = z - \frac{z^{(\delta+1)}}{\delta+1}$$

(A2-3)

Total liquidity generated as a function of z is therefore:

$$y = y_1 + y_2 = f(z) = 1 + \frac{z^{\theta+1}-1}{\theta+1} - \frac{z^{(\delta+1)}}{\delta+1}$$

(A2-4)

Total fire sale costs k resulting from strategy z, k=h(z) are:

$$k = h(z) = \int_z^1 x^\theta \, dx = \frac{1 - z^{(\theta+1)}}{\theta + 1}$$

(A2-5)

This function is monotonously declining in z and can be inverted z = h[-1](k):

$$z = h^{-1}(k) = (1 - (\theta + 1)k)^{\frac{1}{\theta+1}}$$

(A2-6)

Inserting this into y=f(z) generates the monotonous and invertible relationship between liquidity provision and fire sales:

$$y = 1 + \frac{(1 - (\theta + 1)k)^1 - 1}{\theta + 1} - \frac{(1 - (\theta + 1)k)^{\frac{\delta+1}{\theta+1}}}{\delta + 1} = 1 - k - \frac{(1 - (\theta + 1)k)^{\frac{\delta+1}{\theta+1}}}{\delta + 1}$$

(A2-7)

To prove proposition 4, one now has to show that the possibility set (A2-1) dominates the possibility set (A2-7), or in other words that the for a given liquidity k, the former always generates more y, i.e. that for any $k \in [0, 1/(1 + \theta)]$:

$$\frac{\delta}{\delta + 1} + \frac{((\theta + 1)k)^{\frac{\delta+1}{\theta+1}}}{\delta + 1} - k - 1 + k + \frac{(1 - (\theta + 1)k)^{\frac{\delta+1}{\theta+1}}}{\delta + 1} \geq 0$$

(A2-8)

$$<=> \quad \frac{\delta}{\delta + 1} - 1 + \frac{((\theta + 1)k)^{\frac{\delta+1}{\theta+1}} + (1 - (\theta + 1)k)^{\frac{\delta+1}{\theta+1}}}{\delta + 1} \geq 0$$

(A2-9)

$$<=> \quad ((\theta + 1)k)^{\frac{\delta+1}{\theta+1}} + (1 - (\theta + 1)k)^{\frac{\delta+1}{\theta+1}} \geq 1$$

(A2-10)

Note that $(\theta + 1)k \leq 1$ because $k = 1/(\theta + 1)$ is the highest possible level of fire sale losses, namely the one obtained if all assets are fire sold. Second, note that the exponent $(\delta + 1)/(\theta + 1)$ is below one as it was assumed that $\delta < \theta$ (otherwise the trivial case applies that the bank never undertakes any fire sales but always takes recourse to the central bank). So it remains to be shown that for $x\epsilon[0,1]$ and $a\epsilon[0,1]$: $x^a + (1 - x)^a \geq 1$. This is obvious since x^a is a concave function in the unity space. □

Annex 3: Proof of proposition 5

***Proposition 5**: Let $z \in [0,1]$ determine which share of its assets is foreseen by the bank to be used for fire sales (i.e. the less liquid share 1-z of assets are foreseen for pledging with the central bank). Let k=h(z) be the fire sales from fire selling the z most liquid assets and let y=f(z) be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets (1-z). Then a SNNR equilibrium exists if and only if $\exists\, z \in [0,1]$: $y = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1} \geq (1 - t - e)/2$ and $k = h(z) = \frac{z^{(\theta+1)}}{\theta+1} \leq e$.*

First, it has to be shown that a stable unique no-run equilibrium is achieved when (i) liquidity buffers generated by the envisaged strategy are at least as large as the deposits of one short term depositor *and*

(ii) equity buffers are at least as large as to absorb fire sale losses resulting from the strategy. Second, it has to be shown that for a given z, i.e. the share of assets (starting from the most liquid) that is foreseen for fire sales, the liquidity buffers generated are $\frac{\delta}{\delta+1}+\frac{z^{\delta+1}}{\delta+1}-\frac{z^{(\theta+1)}}{\theta+1}$ and fire sale losses are $\frac{z^{(\theta+1)}}{\theta+1}$. Finally, it has to be shown that if $\nexists\, z \in [0,1]: \left(y = f(z) \geq \frac{1-t-e}{2} \cap k = h(z) =\leq e \right)$, then $(U_1(K_1, K_2) \leq U_1(R_1, K_2) \cup U_1(K_1, R_2) > U_1(R_1, R_2)$.

On the first part: Call liquidity buffers available under strategy z y(z) and fire sale losses under strategy z k(z). Assume now that y(z)≥(1-t-e) and e ≥ k(z), then the strategic game between short term depositors takes the following form.

Figure A3-1: The bank run game if y(z)≥(1-t-e) and e ≥ k(z)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε (1-t-e)/2
	Run	(1-t-e)/2 (1-t-e)/2-ε	(1-t-e)/2-ε (1-t-e)/2-ε

This game fulfils the SNNR equilibrium conditions. Now assume that (1-t-e)>y(z)≥(1-t-e)/2 and e ≥ k(z) and also assume (as previously) that e<1/(1+θ). The strategic game then takes the following form. Note that it is assumed below that the central bank will after the counterparty default fire sell the assets pledged with it immediately, which will generate a value as determined by the parameter θ, applied to the (1-z) less liquid part of the assets of the bank. As θ>δ, the central bank will pay back an amount to the insolvency administrator corresponding to the difference. This pay-back will be used to pay out under pari passu (i) the term depositors; (ii) the remaining claims of the short-term depositors. What is essential here is that the recovery ratio RR on these remaining claims of short term depositors will be below one since it was assumed that e < 1/(1+θ). Therefore, the pay-off to the short term depositor will be y(z)/2 + RR ((1-t-e)/2 –y(z)/2) < (1-t-e)/2- ε. Again, the strategic game fulfils the SNNR equilibrium conditions.

Figure A3-2: The bank run game if (1-t-e)>y(z)≥(1-t-e)/2 and e ≥ k(z)

		Depositor II	
		keep deposits	***Run***
Depositor 1	***Keep deposits***	(1-t-e)/2 (1-t-e)/2	(1-t-e)/2-ε (1-t-e)/2
	Run	(1-t-e)/2 (1-t-e)/2-ε	y(z)/2 + RR ((1-t-e)/2 –y(z)/2) y(z)/2 + RR ((1-t-e)/2 –y(z)/2)

Now it has to be shown that for a given z, i.e. the share of assets (starting from the most liquid) that is foreseen for fire sales, the liquidity buffers generated are $\frac{\delta}{\delta+1}+\frac{z^{\delta+1}}{\delta+1}-\frac{z^{(\theta+1)}}{\theta+1}$ and the fire sale losses generated are $\frac{z^{(\theta+1)}}{\theta+1}$. Call y_1 the liquidity that is generated through collateral pledge, and y_2 the liquidity that is generated through fire sales in the strategy described by z. The following figure illustrates the strategy. The central bank liquidity from pledging assets (1-z) starting from the least liquid is:

$$y_1 = (1-z) - \int_z^1 x^\delta \, dx = (1-z) - \left(\frac{1^{\delta+1}}{\delta+1} - \frac{z^{\delta+1}}{\delta+1}\right) = \frac{\delta}{\delta+1} - z + \frac{z^{\delta+1}}{\delta+1}$$

(A3-1)

The liquidity generated from fire selling assets z starting from the most liquid is:

$$y_2 = z - \int_0^z x^\theta \, dx = z - \frac{z^{(\theta+1)}}{\theta+1}$$

(A3-2)

Total liquidity generated as a function of z is therefore:

$$y = y_1 + y_2 = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1}$$

(A3-3)

Total fire sale costs k resulting from strategy z, k=h(z) are:

$$k = h(z) = \int_0^z x^\theta \, dx = \frac{z^{(\theta+1)}}{\theta+1}$$

(A3-4)

Finally, it has to be shown that if $\nexists\, z \in [0,1]: \left(y = f(z) \geq \frac{1-t-e}{2} \cap k = h(z) \leq e \right)$, then $(U_1(K_1, K_2) \leq U_1(R_1, K_2) \cup U_1(K_1, R_2) > U_1(R_1, R_2)$. This can be shown in analogy to similar cases shown in the proof of proposition 2. This completes the proof of Proposition 5. □