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Optimal Climate Policies in a Dynamic Multi-Country Equilibrium Model*

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Abstract

This paper develops a dynamic general equilibrium model with an arbitrary number of different regions to study the economic consequences of climate change under alternative climate policies. Regions differ with respect to their state of economic development, factor endowments, and climate damages and trade on global markets for capital, output, and exhaustible resources. Our main result derives an optimal climate policy consisting of an emissions tax and a transfer policy. The optimal tax can be determined explicitly in our framework and is independent of any weights attached to the interests of different countries. Such weights only determine optimal transfers which distribute tax revenues across countries. We infer that the real political issue is not the tax policy required to reduce global warming but rather how the burden of climate change should be shared via transfer payments between different countries. We propose a simple transfer policy which induces a Pareto improvement relative to the Laissez faire solution.

JEL classification: E10, E61, H21, H23, Q43, Q54

Keywords: Multi-region model; Dynamic equilibrium; Climate change; Optimal climate tax; Optimal transfer policy; Emissions trading system.

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Introduction

Global warming and its consequences for the world economy constitute one of the biggest challenges of the twenty-first century. While the fact that climate change is mainly driven by man-made emissions of carbon-dioxide (CO_2) seems now largely undisputed, political measures to reduce these emissions and limit climate damages remain controversial and are also difficult to implement. An obvious reason for this is that climate change is a *global problem* and climate policies must be coordinated and implemented by different countries with heterogeneous characteristics each acting in their own economic interest which induces well-known problems such as free-riding, etc. cf. Nordhaus (2015).

This general insight suggests that a theoretical evaluation of alternative climate policies should take place in a framework with multiple regions or countries which are politically autonomous and can differ with respect to their economic and other characteristics. The present paper develops such a model to analyze a number of intriguing questions raised in the political debate such as: What is the optimal tax policy to curb emissions? How should this tax differ across regions depending on heterogeneities in economic development, productivity, climate damages, etc.? What is the optimal path of production, emissions, and climate damage in each region and the optimal extraction of fossil resources? Which mix of dirty and clean technologies should each region adopt to produce energy goods and services for production?

Intuitively, one might expect that the answers to these and related questions depend on the weighting of the interests of different regions. Our main result in this paper shows that this intuition is false: All answers to the previous questions are uniquely determined and independent of such weights. In particular, there is no trade-off between the interests of different regions. This insight is based on general structural properties which extend beyond the specific framework employed in this paper. As almost all existing climate models with multiple regions derive optimal climate policies based on (essentially arbitrary) weights assigned to each region (see, e.g., Nordhaus & Yang (1996)), our paper provides a very general theoretical contribution to the existing literature.

Conceptually, the model developed in this paper uses dynamic general equilibrium theory to obtain an internally consistent economic framework with explicit market structures and price formation. The economic part is complemented by a climate model describing how emissions determine atmospheric concentration of carbon dioxide and the resulting damages on the economy. With these features, our model falls into the class of integrated assessment (IA) models which incorporate the full interactions between climate variables and the economic production process.

Most of the existing IA-models are based on the RICE/DICE framework pioneered by Nordhaus (1977) and further developed in Nordhaus & Yang (1996). The numerous extensions and refinements of the RICE/DICE model are comprehensively surveyed in Nordhaus (2011) and, more recently, in Hassler et al. (2016). A typical feature of RICE/DICE-based models stressed in Hassler et al. (2016) is that their solutions are essentially derived as planning problems making only limited use of dynamic general equilibrium theory used in modern macroeconomics. As a consequence, a major problem with the planning approach is the absence of market structures. Among other things, this makes it difficult to study the response of private agents to arbitrary climate policies and, therefore, to evaluate the economic consequences of possibly non-optimal policies.

An alternative framework taking full advantage of dynamic general equilibrium theory is developed in Golosov et al. (2014), henceforth GHKT, who study a single-region model of the world economy. Their main theoretical result is an explicit formula for the optimal tax on fossil emissions. An extension of the GHKT model to a two-region framework is developed in Hassler & Krusell (2012), see also Hassler, Krusell & Olovsson (2010). To preserve analytical tractability, they impose strong restrictions on trade between regions which are only allowed to trade fossil energy inputs.

The framework developed in this paper features an arbitrary number of regions which can differ along various dimensions including factor endowments, productivity, natural resources, and climate damages. Conceptually, it retains the virtues of DSGE modelling from GHKT while allowing for more realistic trade between regions than in Hassler & Krusell (2012). This is accomplished by explicitly separating the *resource stage* at which fossil resources are extracted and the *energy stage* at which energy goods and services are produced. Trade of exhaustible resources is unrestricted in our model while trading of energy outputs is confined to the domestic market. In addition, our model includes an international asset market which permits intertemporal borrowing and lending between regions which is excluded in Hassler & Krusell (2012). While capital is perfectly mobile in our model, labor can only be allocated within each region, which is the traditional Ricardian assumption in trade theory. Modelling the energy stage explicitly also allows us to study how climate policies induce transitions from dirty to clean technologies and affect the energy mix in each region. An extensive numerical study of these questions is offered in a companion paper Hillebrand & Hillebrand (2017).

With the previous features, our model is general enough to incorporate various types of heterogeneity which are potentially important in the political discussion on climate change. At the same time, it remains analytically tractable and permits the derivation of theoretical results. Our main result establishes the existence of an *optimal climate policy* consisting of an *optimal tax* on emissions and a *transfer policy* which determines the distribution of tax revenue across regions via transfers. The optimal tax is uniquely determined and independent of any weights attached to the welfare/utility of different regions. It determines all equilibrium variables except for the distribution of world consumption across regions which depends on the transfer policy. It is this result that led us to claim that the questions raised above can be uniquely answered. Moreover, we can characterize the optimal tax policy explicitly in our model to obtain an intuitive and simple generalization of the tax formula derived in GHKT to a multi-region setting.

Our main result therefore shows that the *efficiency issue* of how to tax emissions can strictly be separated from the *distributional issue* how tax revenue should be shared across countries via transfers to induce a desired distribution of world consumption. Based on this findings, our analysis suggests that the real political issue is not the tax policy required to reduce global warming but rather how the burden of climate change should be shared via transfer payments between different countries. Only the choice of an appropriate transfer scheme induces a trade-off between the interests of different regions and, therefore, should be the subject of negotiations during the political process.

As our final main result, we propose a simple transfer policy under which each region is strictly better off relative to the Laissez faire case where no measures against climate change are taken. Our transfer policy thus satisfies the property of *individual rationality* discussed, e.g., in Eyckmans & Tulkens (2003)) which seems a minimal requirement for a climate policy to be successfully implemented by each region.

The paper is organized as follows. Section 1 introduces the model. The decentralized equilibrium solution and its properties under different climate policies are studied in Section 2. Section 3 studies optimal allocations obtained as solutions to a planning problem. The existence and form of optimal climate policies which implement the optimal solution as an equilibrium allocation is studied in Section 4. Section 5 concludes, mathematical results and proofs can be found in the Appendix.

1 The Model

1.1 World economy

The world economy is divided into $L \geq 1$ regions, indexed by $\ell \in \mathbb{L} := \{1, \ldots, L\}$. Each region $\ell \in \mathbb{L}$ pursues its own interests and takes autonomous political decisions. Although each region typically represent unions or groups of different countries, we will nevertheless refer to region ℓ as a country. Regions are geographically or institutionally separated, which imposes certain restrictions on trade between them.

The production process in each region $\ell \in \mathbb{L}$ decomposes into three stages. The *final sector* produces a consumable output commodity based on a set of inputs including energy goods and services. The second stage consists of a collection of *energy sectors* which produces these goods and services based either on renewable or exhaustible resources. The third stage is represented by the *resource sectors* which extract the domestic stock of exhaustible resources.

These different *production stages* together with a global *climate model* and a description of the *consumption sector* in each region constitute the main building blocks of our model which will be described in detail in the following sections.

1.2 Production sectors

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. There are $I + 1$ production sectors in each region $\ell \in \mathbb{L}$ which are identified by the index $i \in \mathbb{I}_0 := \{0, 1, \ldots, I\}$. Sector $i = 0$ is the *final sector* while $i \in \mathbb{I} := \{1, \ldots, I\}$ identifies the different *energy sectors*. Each sector $i \in \mathbb{I}_0$ consist of a single representative firm which employs labor $N^\ell_{i,t} \geq 0$ and capital $K^\ell_{i,t} \geq 0$ as production factors in period t.

Final sector
Sector $i = 0$ in region $\ell \in \mathbb{L}$ produces final output in period t using the technology

$$Y^\ell_t = (1 - D^\ell_t)Q^\ell_{0,t}F_0(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i\in\mathbb{I}}). \tag{1}$$

Here, $(E^\ell_{i,t})_{i\in\mathbb{I}}$ is as collection of energy inputs used in addition to labor and capital in production. The term $Q^\ell_{0,t} > 0$ in (1) is an exogenous, possibly time- and region-specific productivity parameter which is diminished by damages due to climate change. The latter is measured by a damage index $D^\ell_t \in [0, 1[$ which will be a function of total CO_2 concentration in the atmosphere to be specified below. Finally,

Energy sectors
Energy is supplied by sectors $i \in \mathbb{I}$. Their outputs should be broadly interpreted as energy goods like electricity and heat or services like transportation. We distinguish 'exhaustible' and 'renewable' energy sectors depending on whether they base their production on an exhaustible resource like coal, oil, and natural gas or a renewable resource like wind, water, and solar energy.

Let $\mathbb{I}_x \subset \mathbb{I}$ denote the subset of exhaustible energy sectors. Each such sector $i \in \mathbb{I}_x$ is uniquely identified by the underlying resource on which production is based (like 'coal' used for 'coal-fired power generation' or 'oil' used to provide 'fuel-based transportation services'). The amount of exhaustible resource $i \in \mathbb{I}_x$ used in region $\ell \in \mathbb{L}$ at time t is denoted by $X^\ell_{i,t} \geq 0$. Exhaustible resources are typically an essential input to production in the respective sector and generate emissions proportional to their usage in production. Energy sectors thus represent the production stage at which emissions are potentially generated. Sectors which employ renewable sources do not cause emissions.[1]

With the previous distinction, the technology used by an *exhaustible energy sector* $i \in \mathbb{I}_x$ to produce energy output in period t takes the form

$$E^{\ell,s}_{i,t} = Q^\ell_{i,t}F_i(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}) \tag{2}$$

while production in a *renewable energy sector* $i \in \mathbb{I}\backslash\mathbb{I}_x$ in period t is given by

$$E^{\ell,s}_{i,t} = Q^\ell_{i,t}F_i(K^\ell_{i,t}, N^\ell_{i,t}). \tag{3}$$

[1] This abstracts from emissions generated from using renewable resources like biomass, etc. which are negligible relative to emissions from fossil fuels.

Similar to (1), both specifications (2) and (3) allow for time- and region-specific productivity $Q^{\ell}_{i,t}$. In general, a higher productivity $Q^{\ell}_{i,t} > Q^{\ell'}_{i,t}$ may reflect a more developed technology in region ℓ relative to ℓ' or the fact that conditions to produce energy of type $i \in \mathbb{I}$ are more favorable in region ℓ than in ℓ' due to geographic conditions, etc.[2]

Technology and productivity
Denoting by $\mathbf{Q}_t := (Q^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$ the world productivity vector in period $t \geq 0$, we assume that the evolution of the sequence $(\mathbf{Q}_t)_{t\geq 0}$ is determined exogenously. The remainder imposes the following standard restrictions on production technologies (1), (2), and (3). The Inada condition ensures that each factor is employed in production.

Assumption 1
Each production function $F_i : \mathbb{R}^{n_i}_{+} \longrightarrow \mathbb{R}_{+}$, $i \in \mathbb{I}_0$ is linear homogeneous, strictly increasing, concave, and C^2 on $\mathbb{R}^{n_i}_{++}$. The first partial derivatives satisfy the Inada-condition $\lim_{z_m \searrow 0} \partial_{z_m} F_i(z_1, \ldots, z_{n_i}) = \infty$ $\forall z = (z_1, \ldots, z_{n_i}) \in \mathbb{R}^{n_i}_{++}$ and $m = 1, \ldots, n_i$.

Resource sectors
Resource sectors are uniquely identified by the energy sector $i \in \mathbb{I}_x$ which uses this resource in production. In each region $\ell \in \mathbb{L}$, there exists a single firm which extracts resources of type $i \in \mathbb{I}_x$ and supplies them to the global resource market. The amount of resource i extracted and supplied in period t is denoted $X^{\ell,s}_{i,t} \geq 0$ (to be distinguished from the amount $X^{\ell}_{i,t}$ demanded by energy sector $i \in \mathbb{I}_x$ in that region). Resource firms face constant per unit extraction costs $c_i \geq 0$ and take the initial resource stock $R^{\ell}_{i,0} \geq 0$ as a given parameter.[3] Feasible extraction plans are thus non-negative sequences $(X^{\ell,s}_{i,t})_{t\geq 0}$ which respect the feasibility constraint

$$\sum_{t=0}^{\infty} X^{\ell,s}_{i,t} \leq R^{\ell}_{i,0}. \tag{4}$$

To avoid trivialities, we impose the initial condition $\sum_{\ell\in\mathbb{L}} R^{\ell}_{i,0} > 0$, i.e., initial world resources are strictly positive for all $i \in \mathbb{I}_x$. It may, however, be the case that $R^{\ell}_{i,0} = 0$ in which case region ℓ does not own any resources of type i.

1.3 Climate model

Emissions of CO_2 are generated by using ('burning') exhaustible resources like coal, oil, and gas to produce energy. Thus, emission occur at the energy stage in production. The

[2]For example, a solar energy plant located in the Sahara seems likely to produce more electricity output than an identical plant located in a northern European region like Norway while the opposite holds in the case with hydroelectric power generation.

[3]The assumption of constant extraction costs is a compromise between falling costs due to technological progress assumed in GHKT and extraction costs which increase with the scarcity of the resource as discussed in Hotelling (1931). Our model should be amendable to extensions in either direction.

amount of CO_2 generated by using one unit of exhaustible resource $i \in \mathbb{I}_x$ is physically determined by its carbon-content $\zeta_i \geq 0$. In particular, $\zeta_i = 0$ if the resource does no generate emissions, like uranium in the case of nuclear energy production.[4]

Summing the different types of exhaustible resource inputs weighted by their respective carbon content over all regions one obtains the total emissions of CO_2 in period t as

$$Z_t := \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X^{\ell}_{i,t}. \tag{5}$$

Adopting the specification from GHKT, the climate state in period t consists of permanent and non-permanent CO_2 in the atmosphere and is denoted by $\mathbf{S}_t = (S_{1,t}, S_{2,t})$. Given the sequence of emissions $\{Z_t\}_{t \geq 0}$ determined by (5), the climate state evolves as

$$S_{1,t} = S_{1,t-1} + \phi_L Z_t \tag{6a}$$
$$S_{2,t} = (1-\phi) S_{2,t-1} + (1-\phi_L)\phi_0 Z_t \tag{6b}$$

Specification (6) assumes that a share $0 \leq \phi_L < 1$ of emissions become permanent CO_2. Out of the remaining emissions, a share ϕ_0 becomes non-permanent CO_2 which decays at constant rate $0 < \phi < 1$ while the remaining share $1 - \phi_0$ leaves the atmosphere (see GHKT for details). Total concentration of CO_2 at time t is thus given by

$$S_t = S_{1,t} + S_{2,t}. \tag{7}$$

Climate damages and temperature in period t depend exclusively on S_t. Denoting by $\bar{S} > 0$ the pre-industrial level of CO_2 in the atmosphere, climate damage in region ℓ is determined by a differentiable, strictly increasing function $D^{\ell} : [\bar{S}, \infty[\longrightarrow [0, 1[$,

$$D^{\ell}_t = D^{\ell}(S_t). \tag{8}$$

A specific functional form which will be used below is

$$D^{\ell}(S) = 1 - \exp\{-\gamma^{\ell}(S - \bar{S})\}, \quad \gamma^{\ell} > 0 \tag{9}$$

which corresponds to the choice in GHKT.[5] Reginal differences in climate damage thus enter via region specific parameters γ^{ℓ}, $\ell \in \mathbb{L}$. In particular, the climate problem is economically irrelevant if $\gamma^{\ell} \equiv 0$ (which we do not assume here).

[4]Our specification abstracts from emissions which occur at the resource stage when resources are extracted. While empirically such emissions certainly play a role, especially for uranium, they seem quantitatively negligible compared to emissions occurring at the energy stage on which we focus.

[5]The general version of GHKT allows for γ to be time- and state-dependent. Here, we assume that it is constant, as they do in their numerical simulations, too.

1.4 Consumption sector

The consumption sector in region $\ell \in \mathbb{L}$ consists of a single representative household which supplies labor and capital to the production process and decides about consumption and capital formation taking factor prices as given. In addition, the consumer is entitled to receive all profits from domestic firms and transfers from the government. Let K_0^ℓ denote initial capital in $t = 0$ and $N_t^{\ell,s} > 0$ the labor supplied in period t. The sequence $(\mathbf{N}_t^s)_{t\geq 0}$ of world labor supply $\mathbf{N}_t^s := (N_t^{\ell,s})_{\ell\in\mathbb{L}}$ is exogenously given in our model. The household's preferences over non-negative consumption sequences $(C_t^\ell)_{t\geq 0}$ are represented by a standard time-additive utility function

$$U((C_t^\ell)_{t\geq 0}) = \sum_{t=0}^{\infty} \beta^t u(C_t^\ell). \tag{10}$$

The subsequent analysis imposes the following restrictions on U.

Assumption 2
The discount factor in (10) satisfies $0 < \beta < 1$ *while* u *is of the form*

$$u(C) = \begin{cases} \frac{C^{1-\sigma}}{1-\sigma} & \textit{for} \quad \sigma > 0, \sigma \neq 1 \\ \log(C) & \textit{for} \quad \sigma = 1. \end{cases} \tag{11}$$

Assumption 2 will be key for the separability between efficiency and optimality derived in Section 3 and to determine the optimal climate policy in Section 4. Functional form (11) is precisely the class of utility functions consistent with balanced growth in our model with exogenous labor supply, cf. King, Plosser & Rebelo (1988). Thus, Assumption 2 is a standard restriction in the presence of exogenous productivity growth. It is also used in Nordhaus & Yang (1996) and almost any model of climate change and includes the logarithmic specification in GHKT or Hassler & Krusell (2012) as a special case.

1.5 Summary of the economy

The economy $\mathscr{E}$ introduced in the previous sections can be summarized by its regional and sectoral structure $\langle \mathbb{L}, \mathbb{I}_0, \mathbb{I}_x \rangle$, the production technologies $\langle (\mathbf{Q}_t)_{t\geq 0}, (F_i)_{i\in\mathbb{I}_0}, (c_i)_{i\in\mathbb{I}_x} \rangle$, consumer characteristics $\langle (\mathbf{N}_t^s)_{t\geq 0}, u, \beta \rangle$, and climate parameters $\langle (\zeta_i)_{i\in\mathbb{I}_x}, \phi, \phi_0, \phi_L \rangle$. In addition, initial values for capital supply $\mathbf{K}_0^s = (K_0^\ell)_{\ell\in\mathbb{L}}$, exhaustible resource stocks $\mathbf{R}_0 = (R_{i,0}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x}$, and the initial climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ are given.

2 Decentralized Solution

This section studies the decentralized equilibrium solution of the economy where all producers and consumers behave optimally under perfect foresight and market clearing on all markets. All equilibrium variables are determined for a given *climate policy* which imposes a tax on emissions and distributes the tax revenue as transfers across regions.

2.1 Equilibrium prices

Unless stated otherwise, all prices in period t are denominated in units of time t consumption. As labor and energy outputs will be immobile across countries, their prices will, general, be region-specific. Denote by $w_t^\ell > 0$ the wage and $p_{i,t}^\ell > 0$ the price per unit of energy type $i \in \mathbb{I}$ in region ℓ and period t. By contrast, capital and exhaustible resources are traded on international markets implying that their prices are not country-specific. The (rental) price of capital in period t is denoted as $r_t > 0$ and the world price of resource $i \in \mathbb{I}_x$ as $v_{i,t} > 0$.

Conceptually, all transactions take place in $t = 0$ and the consumption good in this period is chosen as the numeraire. Since the economy is deterministic, the price of time t consumption measured in units of consumption at time zero can be expressed as[6]

$$q_t = \prod_{s=1}^{t} r_s^{-1} \tag{12}$$

for each $t \geq 0$ where $q_0 = 1$. In the following analysis, the price defined in (12) serves as a discount factor which discounts payments in period t to period zero.

2.2 Climate policies

A *climate policy* consists of two parts. The first part is a *Carbon Tax Policy (CTP)* defined next which imposes a proportional tax on CO_2-emissions. Taxes can vary over time and across regions.

Definition 1
A Carbon Tax Policy (CTP) is a non-negative sequence $\tau = (\tau_t)_{t\geq 0}$ where $\tau_t = (\tau_t^\ell)_{\ell\in\mathbb{L}}$ are the (region-specific) taxes to be paid per unit of CO_2 emitted in period $t \geq 0$.

Since emissions occur at the energy stage, taxes are paid by energy producers and redistributed as lump-sum transfers to consumers. A natural restriction would be to assume that transfers equal tax revenue in each region. We will, however, adopt a more general setting which does not require taxes and transfers to balance at the national level but allows for transfer payments across countries. This leads to the following concept of a *transfer policy* which determines the share of tax revenue received by each region. Such a transfer policy constitutes the second part of a climate policy.

[6] In our deterministic model, this holds because $1/r_{t+1}$ is the price of a bond traded in period t that pays-off one unit of the consumption good at time $t + 1$. The prices in (12) are thus the Arrow-Debreu prices for this economy.

Definition 2
A transfer policy is a mapping $\theta : \mathbb{L} \longrightarrow \mathbb{R}$, $\ell \mapsto \theta^\ell$ *satisfying* $\sum_{\ell \in \mathbb{L}} \theta^\ell = 1$ *which determines the share of total tax revenue received by region* $\ell \in \mathbb{L}$ *in each period.*

The pair (τ, θ) will be called a *climate policy*. Let T_t^ℓ denote the transfers received by consumers in region ℓ in period t. These transfers are determined by tax revenue and the given transfer policy as

$$T_t^\ell = \theta^\ell \sum_{k \in \mathbb{L}} \overbrace{\tau_t^k \cdot \underbrace{\sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^k}_{\text{Emissions in region } k}}^{\text{Tax revenue in region } k} \tag{13}$$

If $T_t^\ell > \tau_t^\ell \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell$, region ℓ receives a net transfer from the other countries and contributes a net transfer otherwise. Note that the case $\theta^\ell < 0$ is not excluded in this definition, in which case consumers in region ℓ are taxed to finance transfers received by other countries. Thus, the previous specification also allows for international redistribution via lump-sum taxation. Moreover, the assumption that transfer shares are constant over time is without loss of generality, as the behavior of consumers derived below will exclusively depend on their lifetime transfer income

$$T^\ell := \sum_{t=0}^{\infty} q_t T_t^\ell. \tag{14}$$

Thus, defining total discounted tax revenue

$$T := \sum_{t=0}^{\infty} q_t \sum_{k \in \mathbb{L}} \tau_t^k \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^k, \tag{15}$$

lifetime transfers satisfy

$$T^\ell = \theta^\ell T = \theta^\ell \sum_{t=0}^{\infty} q_t \sum_{k \in \mathbb{L}} \tau_t^k \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^k. \tag{16}$$

and we simply could have defined θ^ℓ as the ratio T^ℓ / T.

2.3 Producer behavior

Firms in each sector $i \in \mathbb{I}_0$ choose their productions plans to maximize the discounted stream of current and future profits. As there is no intertemporal linkage between these decisions, the decision problems of final and energy sectors can be formulated and solved on a period-by period basis. This, however, is not possible in the case of resource sectors which solve intertemporally dependent problems.

Final sector
Given damage-adjusted productivity and factor prices for labor, capital, and the list of energy prices $p_t^\ell = (p_{i,t}^\ell)_{i\in\mathbb{I}} \gg 0$, the final sector in region ℓ solves the following decision problem in period $t \geq 0$:

$$\max_{(K,N,E_1,\dots,E_I)\in\mathbb{R}_+^{2+I}} \left\{(1-D_t^\ell)Q_{0,t}^\ell F_0(K,N,(E_i)_{i\in\mathbb{I}}) - w_t^\ell N - r_t K - \sum_{i\in\mathbb{I}} p_{i,t}^\ell E_i\right\} \tag{17}$$

Under Assumption 1, any solution to (17) satisfies the following first order conditions which equate prices and marginal products of each production factor:

$$(1-D_t^\ell)Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = r_t \tag{18a}$$
$$(1-D_t^\ell)Q_{0,t}^\ell \partial_N F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = w_t^\ell \tag{18b}$$
$$(1-D_t^\ell)Q_{0,t}^\ell \partial_{E_i} F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = p_{i,t}^\ell \quad \forall i \in \mathbb{I}. \tag{18c}$$

Energy sectors
A renewable energy sector $i \in \mathbb{I}\backslash\mathbb{I}_x$ in region $\ell \in \mathbb{L}$ takes sector specific productivity, factor prices for labor and capital, and the domestic energy price $p_{i,t}^\ell > 0$ as given and solves the following decision problem in each period $t \geq 0$:

$$\max_{(K,N)\in\mathbb{R}_+^2} \left\{p_{i,t}^\ell Q_{i,t}^\ell F_i(K,N) - w_t^\ell N - r_t K\right\}. \tag{19}$$

A solution to (19) is characterized by the following first order conditions

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i\big(K_{i,t}^\ell, N_{i,t}^\ell\big) = r_t \tag{20a}$$
$$p_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i\big(K_{i,t}^\ell, N_{i,t}^\ell\big) = w_t^\ell. \tag{20b}$$

An exhaustible energy sector $i \in \mathbb{I}_x$ takes, in addition, the resource price $v_{i,t} > 0$ and the tax $\tau_t^\ell \geq 0$ per unit of CO_2 as given parameters in the decision in period t. As each unit of resource i generates ζ_i units of CO_2, the decision problem in period t reads:

$$\max_{(K,N,X)\in\mathbb{R}_+^3} \left\{p_{i,t}^\ell Q_{i,t}^\ell F_i(K,N,X) - w_t^\ell N - r_t K - (v_{i,t} + \tau_t^\ell \zeta_i)X\right\}. \tag{21}$$

Clearly, the solution to (21) becomes independent of τ_t^ℓ if $\zeta_i = 0$, i.e., the firm employs a clean technology. The first order conditions associated with (21) are given by:

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i\big(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\big) = r_t \tag{22a}$$
$$p_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i\big(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\big) = w_t^\ell \tag{22b}$$
$$p_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i\big(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\big) = v_{i,t} + \zeta_i \tau_t^\ell. \tag{22c}$$

Resource sectors
The resource sector $i \in \mathbb{I}_x$ in region $\ell \in \mathbb{L}$ chooses a non-negative extraction sequence

$(X_{i,t}^{\ell,s})_{t\geq 0}$ which satisfies the resource constraint (4). Resources extracted in period t are supplied to the global resource market. Given sequences of resource prices $(v_{i,t})_{t\geq 0}$, discount factors $(q_t)_{t\geq 0}$ defined by (12) and constant extraction costs $c_i \geq 0$, the resource sector maximizes the discounted sum of future profits. The decision problem reads

$$\max_{(X_{i,t}^{\ell,s})_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} q_t (v_{i,t} - c_i) X_{i,t}^{\ell,s} \mid X_{i,t}^{\ell,s} \geq 0 \; \forall t \geq 0, (4) \text{ holds} \right\}. \tag{23}$$

To avoid trivialities, assume $R_{i,0}^{\ell} > 0$. Then, the existence of optimal extraction plans determined as solutions to problem (23) can be characterized as follows.

Lemma 1
If $R_{i,0}^{\ell} > 0$, the solution to (23) satisfies the following:

(i) An interior solution $(X_t^*)_{t\geq 0} \gg 0$ exists if and only if resource prices satisfy

$$v_{i,t} - c_i = (v_{i,0} - c_i)/q_t \geq 0 \quad \forall t \geq 0. \tag{24}$$

(ii) If condition (24) is satisfied, there are two cases:

(a) If $v_{i,0} = c_i$, any sequence $(X_t^*)_{t\geq 0}$ satisfying $\sum_{t=0}^{\infty} X_t^* \leq R_{i,0}^{\ell}$ is a solution.

(b) If $v_{i,0} > c_i$, any sequence $(X_t^*)_{t\geq 0}$ satisfying $\sum_{t=0}^{\infty} X_t^* = R_{i,0}^{\ell}$ is a solution.

Condition (24) is a version of the classical Hotelling rule (cf. Hotelling (1931)) under which net resource prices must grow at the rate of interest for resource firms to be indifferent between extracting resources in different periods. Hassler & Krusell (2012) also derive a version of the Hotelling rule in their multi-region framework. One also observes from Lemma 1 (ii) that only in case (a) where $v_{i,t} = c_i$ for all $t \geq 0$ may it be optimal not to exhaust the entire stock of resources.
In either case, (24) permits maximum profits $\Pi_i^{\ell} := \sum_{t=0}^{\infty} q_t (v_{i,t} - c_i) X_t^*$ to be written as

$$\Pi_i^{\ell} = (v_{i,0} - c_i) R_{i,0}^{\ell}. \tag{25}$$

Intuitively, the discounted profit stream (25) of resource sector $i \in \mathbb{I}_x$ is the excess value of the initial stock of resources valued at time-zero prices net of extraction costs. Also note that given an optimal extraction plan $(X_{i,t}^{\ell,s})$ determined as a solution to (23), the period profit of resource sector $i \in \mathbb{I}_x$ in region $\ell \in \mathbb{L}$ is given by

$$\Pi_{i,t}^{\ell} = (v_{i,t} - c_i) X_{i,t}^{\ell,s} \geq 0. \tag{26}$$

In general, however, the quantity in (26) will be indeterminate at equilibrium due to the multiplicity of solutions to (23).

Equilibrium profits
All firms in region ℓ are owned by domestic consumers who are entitled to receive all

profits. A direct consequence of Assumption 1 and the first order conditions derived in (18), (20), and (22) is that profits in final production and all energy sectors are zero. Thus, by (25) the total lifetime profit income of consumers in region $\ell \in \mathbb{L}$ is

$$\Pi^\ell = \sum_{i \in \mathbb{I}_x} \Pi_i^\ell = \sum_{i \in \mathbb{I}_x} (v_{i,0} - c_i) R_{i,0}^\ell. \tag{27}$$

2.4 Consumer behavior

Budget constraints

In each period $t \geq 0$, consumers in region $\ell \in \mathbb{L}$ receive labor income $w_t^\ell N_t^{\ell,s} > 0$, the return r_t on their current net asset holdings K_t^ℓ, profit income $\Pi_t^\ell \geq 0$, and transfers T_t^ℓ. Their choices of current consumption $C_t^\ell \geq 0$ and investment K_{t+1}^ℓ satisfy the period budget constraint

$$C_t^\ell + K_{t+1}^\ell = r_t K_t^\ell + w_t^\ell N_t^{\ell,s} + T_t^\ell + \Pi_t^\ell \quad \forall t \geq 0. \tag{28}$$

At the individual level, capital investment may be negative[7] but must satisfy the No-Ponzi game condition

$$\lim_{t \to \infty} q_t K_{t+1}^\ell \geq 0 \tag{29}$$

requiring consumers to ultimately repay any outstanding debt. Using (28) and (29) one can recursively eliminate investment to obtain the consumer's lifetime budget constraint

$$\sum_{t=0}^{\infty} q_t C_t^\ell \leq W^\ell + T^\ell. \tag{30}$$

Here, T^ℓ denotes lifetime transfer income defined in (14) and

$$W^\ell := K_0^\ell + \Pi^\ell + \sum_{t=0}^{\infty} q_t w_t^\ell N_t^{\ell,s} \tag{31}$$

is the consumer's lifetime non-transfer income determined by initial asset holdings K_0^ℓ, lifetime profit income Π^ℓ determined by (27), and lifetime labor income.

Optimal consumption plans

The consumer chooses a consumption sequence $(C_t^\ell)_{t \geq 0}$ to maximize lifetime utility (10) subject to her lifetime budget constraint. The decision problem reads:

$$\max_{(C_t^\ell)_{t \geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t^\ell) \,\middle|\, C_t^\ell \geq 0 \; \forall t \geq 0, (30) \text{ holds} \right\}. \tag{32}$$

[7] This assumption can be justified by assuming that an international bond market coexists with the capital market on which borrowing and lending takes place. The consumer in period t chooses capital investment $\tilde{K}_{t+1}^\ell \geq 0$ and bond purchases B_{t+1}^ℓ subject to the budget constraint (28). Market clearing requires $K_{t+1} = \sum_{\ell \in \mathbb{L}} \tilde{K}_{t+1}^\ell$ on the capital market and $\sum_{\ell \in \mathbb{L}} B_{t+1}^\ell = 0$ for the bond market. Setting $K_{t+}^\ell := \tilde{K}_{t+1}^\ell + B_{t+1}^\ell$ then implies the equilibrium conditions derived below.

Equation (30) shows that existence of a solution to (32) requires the solvency condition

$$T^{\ell} > -W^{\ell} \tag{33}$$

imposing a lower bound on transfers which becomes relevant with taxation ($\theta^{\ell} < 0$). Standard (variational) arguments imply that any solution $(C_t^{\ell *})_{t \geq 0}$ to (32) must satisfy the Euler equations

$$r_{t+1}\beta u'(C_{t+1}^{\ell}) = u'(C_t^{\ell}) \quad \forall t \geq 0. \tag{34}$$

and the lifetime budget constraint (30) holds with equality. The latter is equivalent to the transversality condition

$$\lim_{t \to \infty} q_t K_{t+1}^{\ell} = 0. \tag{35}$$

In fact, the restriction imposed by Assumption 2 allows us to characterize the unique solution to (32) in the following lemma.

Lemma 2
Let Assumption 2 hold and the solvency condition (33) be satisfied. Then, problem (32) has a unique solution $(C_t^{\ell *})_{t \geq 0}$ *given by*

$$C_t^{\ell *} = \frac{(\beta^t/q_t)^{\frac{1}{\sigma}} \left[W^{\ell} + T^{\ell}\right]}{\sum_{s=0}^{\infty} q_s \left(\beta^s/q_s\right)^{\frac{1}{\sigma}}} \qquad t \geq 0. \tag{36}$$

2.5 Market clearing

Restrictions on trade
Trade between countries occurs on global markets for capital, final output, and exhaustible resources of each type $i \in \mathbb{I}_x$. All these goods can freely be exported without additional costs. As there is no sign restriction on capital investment at the individual level, consumers can also take loans permitting intertemporal borrowing and lending of final output between regions. By contrast, labor supply is immobile and can only be employed in domestic production sectors. Likewise, energy goods and services can only be used in domestic final good production.[8] Thus, there are domestic markets for labor and energy of all types $i \in \mathbb{I}$ in each region $\ell \in \mathbb{L}$.

Domestic markets
Market clearing on the domestic labor market in region ℓ in period $t \geq 0$ requires

$$\sum_{i \in \mathbb{I}_0} N_{i,t}^{\ell} \stackrel{!}{=} N_t^{\ell,s}. \tag{37}$$

[8]In fact, this is one reason why we disentangle the energy stage and the resource stage in our model, compared to GHKT.

Since energy is non-tradable across countries, energy demanded in final production must coincide with domestic energy production in each region. The market clearing condition for energy type $i \in \mathbb{I}$ in region $\ell \in \mathbb{L}$ which must hold in each period t is therefore simply

$$E_{i,t}^{\ell} \overset{!}{=} E_{i,t}^{\ell,s}. \tag{38}$$

International markets
Let $K_t := \sum_{\ell \in \mathbb{L}} K_t^{\ell}$ be the aggregate stock of productive capital supplied to production in period t. Market clearing on the world capital market in period t requires

$$\sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_0} K_{i,t}^{\ell} \overset{!}{=} K_t. \tag{39}$$

Since period profit income (26) is, in general, indeterminate at equilibrium, so is consumers' individual net capital position K_t^{ℓ}, $\ell \in \mathbb{L}$ in the period budget constraint (28).

The market clearing condition for exhaustible resource $i \in \mathbb{I}_x$ in period t reads

$$\sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell,s} \overset{!}{=} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell}. \tag{40}$$

If $X_{i,t}^{\ell} < X_{i,t}^{\ell,s}$ region ℓ is a net exporter of resource $i \in \mathbb{I}_x$ in period t and a net importer otherwise. Summing (4) over all countries and using (40), the allocation of resources across production sectors must satisfy the world resource constraint

$$\sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell} \leq R_{i,0} \tag{41}$$

where $R_{i,0} := \sum_{\ell \in \mathbb{L}} R_{i,0}^{\ell}$ is the total initial stock of resource i. An immediate consequence of Lemma 1 is that the equilibrium amount of resources $X_{i,t}^{\ell,s}$ supplied by region ℓ and period profits (26) will, in general, be indeterminate. As resources extracted in different countries are perfect substitutes, however, the equilibrium extraction plans can always be chosen compatible with the resource constraint (4) in each region.

Finally, summing the consumers' period budget constraints (28) over all regions and exploiting the first order and zero profit conditions for all sectors $i \in \mathbb{I}_0$ in conjunction with (26) and the market clearing conditions (37), (38), (39), and (40) together with (13) one obtains the market clearing condition for final output in period t as

$$K_{t+1} + \sum_{\ell \in \mathbb{L}} C_t^{\ell} + \sum_{i \in \mathbb{I}_x} c_i \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell} = \sum_{\ell \in \mathbb{L}} Y_t^{\ell}. \tag{42}$$

Here, K_{t+1} is productive capital formed in period t and supplied to production in $t+1$.

2.6 Equilibrium

For purposes of a compact notation, we employ the following vector notation for the variables introduced in the previous sections for each t:

$$\begin{array}{lllllllll}\mathbf{Q}_t & := & (Q^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{N}^s_t & := & (N^{\ell,s}_t)_{\ell\in\mathbb{L}} & \mathbf{C}_t & := & (C^{\ell}_t)_{\ell\in\mathbb{L}} \\ \mathbf{Y}_t & := & (Y^{\ell}_t)_{\ell\in\mathbb{L}} & \mathbf{E}_t & := & (E^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} & \mathbf{X}_t & := & (X^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \\ \mathbf{N}_t & := & (N^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{K}_t & := & (K^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{S}_t & := & (S_{t,1}, S_{t,2}) \\ \mathbf{w}_t & := & (w^{\ell}_t)_{\ell\in\mathbb{L}}, & \mathbf{p}_t & := & (p^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} & \mathbf{v}_t & := & (v_{i,t})_{i\in\mathbb{I}_x}.\end{array} \tag{43}$$

All variables defined in (43) take values in the appropriate positive orthant of $\mathbb{R}^n$.

Definition of equilibrium
Let the climate tax policy τ and transfer policy θ defined as above be given. The following definition of equilibrium is standard. Here and in the remainder we denote equilibrium variables by a * superscript.

Definition 3
Given tax policy τ and transfer policy θ, an equilibrium of $\mathscr{E}$ is an allocation $\mathbf{A}^* = \big(\mathbf{C}^*_t, K^*_{t+1}, \mathbf{Y}^*_t, \mathbf{E}^*_t, \mathbf{K}^*_t, \mathbf{N}^*_t, \mathbf{X}^*_t, \mathbf{S}^*_t\big)_{t\geq 0}$ and prices $\mathbf{P}^* = \big(r^*_t, \mathbf{w}^*_t, \mathbf{p}^*_t, \mathbf{v}^*_t\big)_{t\geq 0}$ such that:

(i) The allocation is consistent with the production technologies (1), (2), and (3) and the market clearing conditions/resource constraints (37), (38), (39), (41), and (42).

(ii) Producers behave optimally, i.e., equations (18), (22), and (20) hold for all $t \geq 0$. Profits of resource firms are given by (25) while resource prices evolve as in (24).

(iii) Consumers behave optimally as described in Lemma 2 with profit incomes determined by (27) and transfers satisfying (16) and the solvency condition (33).

(iv) Climate variables evolve according to (6) with emissions given by (5) and climate damages in (1) determined by (7) and (8).

Properties of equilibrium
If we want to emphasize the dependence of the equilibrium allocation on policy variables, we will write $\mathbf{A}^*(\tau, \theta)$, etc. A special case of Definition 3 is the Laissez faire equilibrium with no taxation, i.e., $\tau \equiv 0$. The induced equilibrium allocation $\mathbf{A}^{\mathrm{LF}} := \mathbf{A}^*(0, \theta)$ is independent of θ and constitutes an important benchmark in the subsequent discussion.[9] It is clear that this solution will, in general not constitute a Pareto optimal outcome due to the climate externality in production.

[9] Our subsequent discussion and notation presume that each policy considered induces a unique equilibrium. A general proof of existence/uniqueness of equilibrium is beyond the scope of this paper. The recursive structure of equilibria and their computation is discussed in detail in Hillebrand & Hillebrand (2017) suggesting that the uniqueness assumption is justified.

The following results establish additional properties of equilibrium that follow from our restrictions on technologies and preferences. As before, we denote by $R_{i,0} = \sum_{\ell \in \mathbb{L}} R^{\ell}_{i,0}$ to be the total initial stock of resource $i \in \mathbb{I}_x$.

Lemma 3

Under Assumptions 1 and 2, any equilibrium allocation has the following properties:

(i) The allocation is interior, i.e., $\mathbf{A}^* \gg 0$.

(ii) Consumption $C^{\ell *}_t$ *is a constant share of world consumption* $\bar{C}^*_t := \sum_{\ell \in \mathbb{L}} C^{\ell *}_t$, *i.e.,*

$$C^{\ell *}_t = \mu^{\ell *} \bar{C}^*_t \tag{44}$$

for all $\ell \in \mathbb{L}$ *and* $t \geq 0$ *where* $\mu^{\ell *} > 0$ *and* $\sum_{\ell \in \mathbb{L}} \mu^{\ell *} = 1$.

(iii) Prices and extraction of each resource $i \in \mathbb{I}_x$ *satisfy the following:*

(a) If $R_{i,0} = \infty$, *then* $v_{i,0} = c_i$ *implying* $v_{i,t} \equiv c_i$.

(b) If $R_{i,0} < \infty$, *then* $\lim_{t \to \infty}(v_{i,t} + \zeta_i \tau^{\ell}_t) = \infty$.

(c) If $R_{i,0} > \sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X^{\ell *}_{i,t}$, *then* $v_{i,0} = c_i$ *and* $\lim_{t \to \infty}(v_{i,t} + \zeta_i \tau^{\ell}_t) = \infty$.

Lemma 3 (ii) shows that consumption in each region moves in lock-step with world consumption over time. This property is due to consumer utility restricted by Assumption 2 and will play a key role to characterize how policy affects equilibrium allocations.

The result in (iii) shows that an infinite stock of resources drives resource prices down to extraction costs. Hence, there is no scarcity rent on that resource and profits are zero.[10] Conversely, if the resource stock is finite, gross resource prices (including taxes) must converge to infinity which requires $v_{i,0} > c_i$ in the absence of taxation. Moreover, (iii) shows that each resource is completely exhausted unless its scarcity rent is zero and taxes on emissions grow infinitely large. Thus, aggressive taxation is needed to prevent resources from being completely exhausted. In particular, all resources are fully exhausted at the Laissez faire equilibrium. See Hillebrand & Hillebrand (2017) for further discussion and application of these results in a numerical study of the model.

The final result of this section disentangles the separate impact of the tax policy τ and the transfer policy θ on the equilibrium allocation $\mathbf{A}^*(\tau, \theta)$. Essentially, it shows that transfers only determine consumption in each region as a constant share of world consumption but are irrelevant for all other equilibrium variables. This result will play a major role in Section 4 when we study climate policies which implement the social optimum as an equilibrium allocation.

[10] An infinite resource stock is typically justified by assuming the existence of a backstop technology which provides an equivalent substitute for the resource in the future. Such a backstop technology is implicitly assumed in GHKT for coal.Hillebrand & Hillebrand (2017) provide a critical discussion of this asssumption and show that it is key to obtain the quantitative predictions in GHKT.

Proposition 1
Given a climate policy (τ, θ)*, let* $\mathbf{A}^* = \left(\mathbf{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*\right)_{t\geq 0}$ *be the induced equilibrium allocation and* $\mathbf{P}^*$ *the equilibrium price system. For each* $t \geq 0$*, let* $\bar{C}_t^* = \sum_{\ell\in\mathbb{L}} C_t^{\ell*}$ *denote aggregate equilibrium consumption. Then, the following holds:*

(i) The tax policy τ *determines the aggregate equilibrium allocation*

$$\bar{\mathbf{A}}^* = \left(\bar{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*\right)_{t\geq 0} \tag{45}$$

and the price system $\mathbf{P}^*$ *which are both independent of the transfer policy* θ*.*

(ii) The transfer policy θ *only affects the distribution of aggregate consumption across regions, i.e., the consumption shares* $\mu^{\ell*}$ *in (44) which take the form*

$$\mu^{\ell*} = \frac{W^{\ell*} + \theta^\ell T^*}{\sum_{k\in\mathbb{L}} W^{k*} + T^*}. \tag{46}$$

with non-transfer incomes $W^{\ell*}$ *determined by (31) and tax revenue* T^* *by (15).*

The aggregate allocation $\bar{\mathbf{A}}^*$ in (45) determines world consumption but does not specify its distribution across regions. Apart from that, it contains the same variables as the equilibrium allocation $\mathbf{A}^*$. The equilibrium distribution of consumption determined by (46) corresponds to the relative sizes of consumers' lifetime incomes including transfers. Only this equilibrium quantity depends on the transfer policy. Note that consumption shares become independent of θ along the Laissez faire equilibrium where $T^* = 0$.

3 Optimal Solution

In this section we determine an *optimal allocation* as the solution to a planning problem (PP) which maximizes consumer utility subject to the feasibility constraints imposed by technology, resources, and climate change. The major difference to the decentralized solution is that the planning problem incorporates the climate externality and the link between emissions, climate damage, and productivity in final good production.

In a multi-region world, there is no unique choice of the planner's objective function which must necessarily incorporate the trade-offs between the interests of different countries. The standard approach in the literature (see, eg., Nordhaus & Yang (1996)) also adopted here aggregates utilities in different countries based on a weighting scheme which assigns a certain weight to the utility of consumers in each region. Such a weighting scheme is essentially equivalent to choosing certain minimum utility levels for all countries $\ell \neq 1$ and then maximizing utility of region 1 as is done in Eyckmans & Tulkens (2003) to obtain a Pareto-optimal allocation.

A major advantage of our restrictions on preferences in Assumption 2 is that it permits to compute an optimal allocation in two steps. First, we determine an *efficient allocation* which maximizes utility of a fictitious world representative consumer. This efficient solution completely specifies the optimal climate path and the entire allocation of production factors and resources across countries together with aggregate world consumption. Most importantly, the efficient solution is *independent* of the employed weighting scheme. In a second step, we determine an *optimal distribution* of world consumption across different countries to obtain an *optimal allocation* which maximizes a weighted utility index reflecting the trade-off between the interests of different countries. This separability between efficiency and optimal distribution will be the key to determine an optimal climate policy in the next chapter.

3.1 Optimal allocations

Feasible allocations
Consider a planner who chooses a feasible world allocation subject to the restrictions imposed by technology, factor mobility, and resource constraints. Formally, using the notation introduced in the previous section, the planner takes the sequences of productivity $(\mathbf{Q}_t)_{t\geq 0}$ and labor supply $(\mathbf{N}_t^s)_{t\geq 0}$ as given. In addition, initial world capital $K_0 > 0$, the initial world stock $R_{i,0} > 0$ of each exhaustible resource $i \in \mathbb{I}_x$, and the initial climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ are given as well. While final output, capital, and exhaustible resources can freely be allocated across countries, labor and energy outputs can only be used within each region. Thus, the planner faces essentially the same restrictions as producers and consumers in the decentralized solution including constraints (37), (38), (39), and (42) when allocating labor, capital, and output. Further, the given initial stock of world resources imposes the restrictions (41) on the use of exhaustible resource $i \in \mathbb{I}_x$ in production.[11] This leads to the following definition of a feasible allocation.

Definition 4

(i) A feasible allocation is a sequence $\mathbf{A} = \big(\mathbf{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t\big)_{t\geq 0}$ *which satisfies (1), (2), (3), (5), (6), (7), (37), (38), (39), (41), and (42) for all* $t \geq 0$.

(ii) The set of feasible allocations of the economy $\mathscr{E}$ *is denoted* $\mathbb{A}$.

In particular, equilibrium allocations studied in the previous section are feasible, i.e., $\mathbf{A}^* \in \mathbb{A}$.

[11] As resources extracted in different countries are perfect substitutes, the solution to the planning problem does not determine where these resources are extracted. However, any allocation of exhaustible resources $(X_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x})_{t\geq 0}$ satisfying the feasibility constraint (41) can always be chosen compatible with the individual resource constraints (4) in each region.

Objective function
The distribution of consumption across countries necessarily induces a trade-off between the utility levels attained by consumers in different countries. To incorporate this trade-off, assume that the planner uses a weighting scheme corresponding to a list of utility weights $\omega = (\omega^\ell)_{\ell \in \mathbb{L}}$ where $\omega^\ell \geq 0$ represents the weight attached to consumer utility in region ℓ in the planner's decision. Formally, we have

Definition 5
A utility weighting scheme is a map $\omega : \mathbb{L} \longrightarrow \mathbb{R}_+$, $\ell \mapsto \omega^\ell$ *which satisfies* $\sum_{\ell \in \mathbb{L}} \omega^\ell = 1$.

In what follows, let $\Delta := \{(x_1, \ldots, x_L) \in \mathbb{R}^L | \sum_{\ell=1}^{L} x_\ell = 1\}$ denote the unit-simplex in $\mathbb{R}^L$. Then, the set of all possible weighting schemes can be identified with $\Delta_+ := \Delta \cap \mathbb{R}^L_+$. Each weighting scheme $\omega \in \Delta_+$ defines the following weighted utility index

$$V((\mathbf{C}_t)_{t \geq 0}; \omega) := \sum_{t=0}^{\infty} \beta^t \sum_{\ell \in \mathbb{L}} \omega^\ell u(C_t^\ell) \tag{47}$$

which depends on world consumption $(\mathbf{C}_t)_{t \geq 0}$ where $\mathbf{C}_t = (C_t^\ell)_{\ell \in \mathbb{L}}$ for all $t \geq 0$.

Weighted Planning Problem
Given $\omega \in \Delta_+$, use (47) to define the following **Weighted Planning Problem (WPP)**

$$\max_{\mathbf{A}} \left\{ V((\mathbf{C}_t)_{t \geq 0}; \omega) \middle| \, \mathbf{A} = (\mathbf{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0} \in \mathbb{A} \right\}. \tag{48}$$

Assuming it exists and is unique, denote the solution to (48) as

$$\mathbf{A}^{\text{opt}} = \Big(\mathbf{C}_t^{\text{opt}}, K_{t+1}^{\text{opt}}, \mathbf{Y}_t^{\text{opt}}, \mathbf{E}_t^{\text{opt}}, \mathbf{K}_t^{\text{opt}}, \mathbf{N}_t^{\text{opt}}, \mathbf{X}_t^{\text{opt}}, \mathbf{S}_t^{\text{opt}})_{t \geq 0}. \tag{49}$$

It is clear that, in general, the solution to (48) will depend on the weighting scheme ω. Thus, we will write $\mathbf{A}^{\text{opt}}(\omega)$ as a way of emphasizing this dependence. It is also clear that for any weighting scheme $\omega \in \Delta_+$ the solution $\mathbf{A}^{\text{opt}}(\omega)$ to (48) is a Pareto-optimum on the set of feasible allocations $\mathbb{A}$.

3.2 Efficient aggregate allocations

Feasible aggregate allocations
Consider now a modified planning problem which faces the same restrictions as before but does not specify the distribution of consumption across different countries. As before, denote aggregate world consumption in period t by $\bar{C}_t \geq 0$ and write the resource constraint (42) as

$$K_{t+1} + \bar{C}_t + \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} c_i X_{i,t}^\ell = \sum_{\ell \in \mathbb{L}} Y_t^\ell. \tag{50}$$

Replacing (42) by (50) leads to the following definition of a feasible aggregate allocation which specifies aggregate world consumption but not its distribution across countries. Apart from that, it involves the same variables as a feasible allocation defined above.

Definition 6

(i) A feasible aggregate allocation is a sequence $\bar{\mathbf{A}} = \big(\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t\big)_{t\geq 0}$ *which satisfies (1), (2), (3), (5), (6), (7), (37), (38), (39), (41), (50) for all* $t \geq 0$*.*

(ii) The set of feasible aggregate allocations of the economy $\mathscr{E}$ *is denoted* $\bar{\mathbb{A}}$*.*

In particular, the aggregate equilibrium allocation in (45) is feasible, i.e., $\bar{\mathbf{A}}^* \in \bar{\mathbb{A}}$.

Aggregate planning problem

The modified planning problem maximizes utility of a fictitious world representative consumer who consumes $\bar{C}_t$ in period t and has the same utility function as consumers in each region. This leads to the following **Aggregate Planning Problem (APP)**:

$$\max_{\bar{\mathbf{A}}} \left\{ \sum_{t=0}^{\infty} \beta^t u(\bar{C}_t) \,\middle|\, \bar{\mathbf{A}} = (\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t\geq 0} \in \bar{\mathbb{A}} \right\}. \tag{51}$$

A solution to (51) will be denoted

$$\bar{\mathbf{A}}^{\text{eff}} = (\bar{C}_t^{\text{eff}}, K_{t+1}^{\text{eff}}, \mathbf{Y}_t^{\text{eff}}, \mathbf{E}_t^{\text{eff}}, \mathbf{K}_t^{\text{eff}}, \mathbf{N}_t^{\text{eff}}, \mathbf{X}_t^{\text{eff}}, \mathbf{S}_t^{\text{eff}})_{t\geq 0} \tag{52}$$

and referred to as an *efficient aggregate allocation*. An efficient solution $\bar{\mathbf{A}}^{\text{eff}}$ specifies the entire allocation of production factors and resources across countries but leaves undetermined the distribution of aggregate consumption across countries. It is therefore independent of any weights attached to the interests of different countries and constitutes the main ingredient to our separation result.

3.3 Optimal distribution of consumption

To prepare the main result of this section, let a weighting scheme $\omega \in \Delta_+$ be arbitrary but fixed and consider the static problem of distributing a given level $\bar{C} > 0$ of aggregate consumption across countries in an arbitrary period. The decision variable is a *consumption distribution* $\mu = (\mu^\ell)_{\ell\in\mathbb{L}}$ where $\mu^\ell \geq 0$ is the share of $\bar{C}$ given to region ℓ. Since $\sum_{\ell\in\mathbb{L}} \mu^\ell = 1$, any feasible consumption distribution satisfies $\mu \in \Delta_+$ defined as above. An optimal consumption distribution $\mu^{\text{opt}} = (\mu^{\ell,\text{opt}})_{\ell\in\mathbb{L}}$ can be determined as the solution to the problem

$$\max_{\mu=(\mu^\ell)_{\ell\in\mathbb{L}}} \left\{ \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell \bar{C}) \,\middle|\, \mu^\ell \geq 0 \; \forall \ell \in \mathbb{L}, \sum_{\ell\in\mathbb{L}} \mu^\ell \leq 1 \right\}. \tag{53}$$

The following lemma establishes that (53) has a unique solution which can be computed explicitly and which, crucially, is independent of $\bar{C}$. The result in (ii) establishes an essential equivalence between the choice of a weighting scheme ω and a consumption distribution μ which will be exploited in Section 4.3.

Lemma 4
Let u be of the form (11) from Assumption 2. Then, the following holds:

(i) For each weighting scheme $\omega = (\omega^\ell)_{\ell\in\mathbb{L}} \in \Delta_+$, there exists a unique consumption distribution $\mu^{\mathrm{opt}}(\omega) = (\mu^{\ell,\mathrm{opt}}(\omega))_{\ell\in\mathbb{L}}$ which solves (53) taking the form

$$\mu^{\ell,\mathrm{opt}}(\omega) = \frac{(\omega^\ell)^{\frac{1}{\sigma}}}{\sum_{k\in\mathbb{L}}(\omega^k)^{\frac{1}{\sigma}}}, \quad \ell \in \mathbb{L}. \tag{54}$$

(ii) For any consumption distribution $\mu = (\mu^\ell)_{\ell\in\mathbb{L}} \in \Delta_+$ there exists a weighting scheme $\omega \in \Delta_+$ which rationalizes μ in the sense that $\mu = \mu^{\mathrm{opt}}(\omega)$ solves (53).

3.4 From efficiency to optimality

The main result of this section shows that the solution (49) to the WPP (48) can be obtained from the efficient allocation (52) by distributing aggregate consumption optimally across countries. Moreover, for any weighting scheme ω, the optimal distribution is time-invariant and can be determined as the solution to (53). Using Lemma 4 allows us to state this result in the following theorem.

Theorem 1
Let Assumptions 1 and 2 be satisfied and define the efficient allocation $\mathbf{A}^{\mathrm{eff}}$ as in (52). Given a weighting scheme ω, let $\mu^{\mathrm{opt}}(\omega) = (\mu^{\ell,\mathrm{opt}}(\omega))_{\ell\in\mathbb{L}}$ solve (53). Then, the allocation

$$\mathbf{A} = (\mu^{\mathrm{opt}}(\omega)\bar{C}_t^{\mathrm{eff}}, K_{t+1}^{\mathrm{eff}}, \mathbf{Y}_t^{\mathrm{eff}}, \mathbf{E}_t^{\mathrm{eff}}, \mathbf{K}_t^{\mathrm{eff}}, \mathbf{N}_t^{\mathrm{eff}}, \mathbf{X}_t^{\mathrm{eff}}, \mathbf{S}_t^{\mathrm{eff}})_{t\geq 0} \tag{55}$$

solves the WPP (48), i.e., $\mathbf{A} = \mathbf{A}^{\mathrm{opt}}(\omega)$.

In words, Theorem 1 says that a solution to the WPP can simply be obtained from the efficient solution (52) by distribution aggregate consumption in an optimal fashion determined by the weighting scheme ω. In particular, the entire allocation of production factors, resources, and emissions is completely determined by the efficient solution.

As its major implication, the previous result allows to compute a unique efficient allocation of production factors, resources, emissions, climate damages, etc. which is completely independent of the weights that the interests of different countries receive in the decision. The weighting scheme is therefore irrelevant for answering the question what the optimal climate path is and *where* and *how* emissions should be reduced. Intuitively, there exists a unique allocation of factors and resources across regions to produce an efficient world consumption sequence which incorporates the climate externality. The weighting scheme becomes relevant only to determine how this efficiently produced world consumption sequence should be distributed across regions. An optimal consumption distribution can be computed using (54) once a suitable weighting scheme has been chosen and, by Lemma 4 (ii) is in fact equivalent to such a choice.

3.5 Computing the efficient allocation

The previous results show that the social optimum is essentially characterized by the efficient solution (52). Adopting a standard infinite-dimensional Lagrangian approach, it is now straightforward to obtain explicit conditions which completely characterize this solution. Detailed computations can be found in Section A.7 in the appendix. The main findings are as follows.

Cost of climate damage

The total costs Λ_t of emitting one additional unit of CO_2 in period t (measured in units of time t consumption) equals the discounted sum of all future marginal climate damages in all regions caused by this emission. Formally,

$$\Lambda_t = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n \Big) \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_{t+n})}{dS} \frac{Y^\ell_{t+n}}{1-D^\ell(S_{t+n})}. \tag{56}$$

Equation (56) is a multi-region version of the result in GHKT. Note that Λ_t is independent of ℓ and i and depends on the structural parameters of the model and endogenous model variables in a complicated way, unless stronger restrictions similar to those in GHKT are imposed.[12] The term (56) is the key quantity to incorporate the climate externality into the (shadow) price of exhaustible resources.

Efficiency conditions

The remaining optimality conditions essentially ensure *intratemporal* and *intertemporal efficiency* in production in each period $t \geq 0$. Denote by

$$\hat{p}^\ell_{i,t} = (1-D^\ell_t) Q^\ell_{0,t} \partial_{E_i} F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, \mathbf{E}^\ell_t\big). \tag{57}$$

the time t *shadow price* of energy type $i \in \mathbb{I}$ in region $\ell \in \mathbb{L}$ (measured in units of time t consumption units). Using (57), marginal products of capital are equalized across all countries and sectors, i.e., for all $\ell, \ell' \in \mathbb{L}$ and all $i \in \mathbb{I}$:[13]

$$(1-D^\ell(S_t)) Q^\ell_{0,t} \partial_K F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, \mathbf{E}^\ell_t\big) = \hat{p}^{\ell'}_{i,t} Q^{\ell'}_{i,t} \partial_K F_i(K^{\ell'}_{i,t}, N^{\ell'}_{i,t}, X^{\ell'}_{i,t}) \tag{58}$$

Second, in each region $\ell \in \mathbb{L}$ marginal products of labor are equalized across all sectors (although not across countries due to labor immobility), i.e., for all $i \in \mathbb{I}$

$$(1-D^\ell(S_t)) Q^\ell_{0,t} \partial_N F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i\in\mathbb{I}}\big) = \hat{p}^\ell_{i,t} Q^\ell_{i,t} \partial_N F_i(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}). \tag{59}$$

Third, for all $i \in \mathbb{I}_x$, the constraint (41) is binding and resource extraction is intratemporally efficient in each period $t \geq 0$, i.e., for all $\ell \in \mathbb{L}$:

$$\hat{p}^\ell_{i,t} Q^\ell_{i,t} \partial_X F_i(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}) - \zeta_i \Lambda_t = \hat{v}_{i,t}. \tag{60}$$

[12] Climate costs Λ_t are independent of ℓ because future climate damages are discounted by the same discount factors in each region. For a different setup with region-specific discount rates see Eyckmans & Tulkens (2003).

[13] Here and in the sequel, we incur a slight abuse of notation by including $X^\ell_{i,t}$ as a 'dummy' argument of F_i even if $i \notin \mathbb{I}_x$ to save some notation.

Equation (60) defines the true shadow price of resources as the marginal product in production minus the cost of emissions defined in (56). Compared to the laissez-faire equilibrium allocation, the social planner includes a wedge between the marginal product of dirty resource i and its (shadow) price which accounts for the externality cost of an additional unit of emissions. This is the key difference to the equilibrium condition (22c) along the Laissez faire equilibrium which fails to take this cost into account. If $D^\ell \equiv 0$, however, the efficient solution coincides with the aggregate equilibrium allocation (45).

Intertemporal efficiency of final good allocation (consumption vs. capital formation) is ensured by the standard Euler equation which holds for all $\ell \in \mathbb{L}$ and $t \geq 1$:

$$u'(\bar{C}_{t-1}) = \beta u'(\bar{C}_t)(1 - D^\ell(S_t))Q^\ell_{0,t}\partial_K F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i\in\mathbb{I}}\big). \tag{61}$$

Condition (61) in conjunction with (58) also equates (implicit) capital returns across countries in each period.

Defining the shadow price of resource extraction as in (60), intertemporally efficient extraction of resource $i \in \mathbb{I}_x$ is ensured by the condition:

$$\hat{v}_{i,t} - c_i = \frac{\beta u'(\bar{C}_{t+1})}{u'(\bar{C}_t)}\Big(\hat{v}_{i,t+1} - c_i\Big). \tag{62}$$

Finally, standard arguments also require the transversality condition

$$\lim_{t\to\infty} \beta^t u'(\bar{C}_t)K_{t+1} = 0. \tag{63}$$

The following proposition summarizes the result of this section. The proof is given in Section A.7 in the appendix.

Proposition 2
Let Assumptions 1 and 2 hold. Then, any feasible aggregate allocation $\bar{\mathbf{A}} \in \bar{\mathbb{A}}$ which satisfies conditions (56)-(62) for all $t \geq 0$ as well as (63) is efficient, i.e., solves (51).

4 Optimal Climate Policies

This section determines a climate policy which implements the ω-optimal allocation (49) as an equilibrium allocation in the sense of Definition 3. Such a policy will be referred to as an *optimal climate policy.* The properties of equilibria stated in Proposition 1 and the separation result from Section 3 allow us to derive the optimal policy in two steps. In the first step, an efficient climate tax policy τ^{eff} is computed which implements the efficient allocation (52) as an aggregate equilibrium allocation defined as in (45), i.e., $\bar{\mathbf{A}}^*(\tau^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$. In the second step, an optimal transfer scheme $\theta^{\text{opt}}(\omega)$ is computed based on a given weighting scheme ω which together with τ^{eff} implements the ω-optimal allocation as an equilibrium allocation, i.e., $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{opt}}(\omega)) = \mathbf{A}^{\text{opt}}(\omega)$. As the latter requires the choice of a weighting scheme ω, we offer a simple idea how such a scheme could be chosen such that each region has an incentive to adopt the optimal climate tax policy.

4.1 The optimal climate tax policy

Using the efficient allocation (52), define the climate tax policy $\tau^{\text{eff}} = (\tau_t^{\text{eff}})_{t \geq 0}$ as

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n}^{\text{eff}})}{u'(\bar{C}_t^{\text{eff}})} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n \Big) \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_{t+n}^{\text{eff}})}{dS} \frac{Y_{t+n}^{\ell,\text{eff}}}{1 - D^\ell(S_{t+n}^{\text{eff}})}, \quad t \geq 0. \tag{64}$$

Equation (64) is a classical example of a *Pigovian tax* which equates taxes to the total discounted marginal damage caused by each unit of CO_2 determined by (56). Under this policy, emissions taxes $\tau_t^\ell \equiv \tau_t^{\text{eff}}$ are uniform across dirty sectors in all countries and incorporate the total damage from emitting one unit of CO_2 in period t. The following result shows that this policy implements the efficient solution (52) as an aggregate equilibrium allocation defined as in (45). Recall from Proposition 1 that this allocation is independent of the transfer policy. Thus, the CTP defined by (64) will be referred to as the *efficient tax policy*.

Theorem 2
Let Assumptions 1 and 2 hold and define the climate tax policy τ^{eff} as in (64). Then, the induced aggregate equilibrium allocation defined in (45) is efficient, i.e., $\bar{\mathbf{A}}^*(\tau^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$.

With the specific functional forms (9) and (11) for climate damage and consumer utility, the efficient tax formula (64) takes the more specific form

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \left(\frac{\bar{C}_{t+n}^{\text{eff}}}{\bar{C}_t^{\text{eff}}} \right)^{-\sigma} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n \Big) \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_{t+n}^{\ell,\text{eff}}. \tag{65}$$

In particular, efficient taxes are zero if climate damages are absent, i.e., $\gamma^\ell \equiv 0$.

In general, expression (65) can not be computed explicitly, as it involves the entire paths of future output in each region and aggregate consumption. However, if the efficient solution induces a balanced growth path on which output and consumption grow at constant and identical rate $g \geq 0$, (65) takes the much simpler form

$$\tau_t^{\text{eff}} = \bar{\tau}^{\text{eff}} \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_t^{\ell,\text{eff}}, \quad \bar{\tau}^{\text{eff}} := \frac{\phi_L}{1 - \beta(1+g)^{1-\sigma}} + \phi_0 \frac{1-\phi_L}{1 - \beta(1+g)^{1-\sigma}(1-\phi)}. \tag{66}$$

Thus, on a balanced growth path, the optimal tax is a constant share $\bar{\tau}^{\text{eff}}$ of world output weighted by the damage parameters γ^ℓ. For logarithmic utility ($\sigma = 1$) and homogeneous climate damages ($\gamma^\ell \equiv \gamma$), equation (66) recovers precisely the tax-formula derived in

GHKT under a set of additional restrictions (log utility, Cobb-Douglas production, all capital used in the final sector). None of these restrictions is required here if the assumption of a balanced growth path is satisfied. Numerical simulations of the model presented in Hillebrand & Hillebrand (2017) show that the efficient solution converges quickly to a balanced growth path suggesting that (65) is well-approximated by (66) in applications of the model.

4.2 Optimal transfer policies

Under the efficient climate tax policy (64), the aggregate equilibrium allocation (45) coincides with the efficient allocation (52). While this determines aggregate world consumption together with an optimal allocation of production factors and climate variables in each period, it leaves undetermined how world consumption is distributed across countries. The latter requires the choice of a weighting scheme $\omega \in \Delta_+$ based on which the optimal consumption distribution $\mu^{\mathrm{opt}}(\omega)$ can be determined by (53).

We now explore the existence of a transfer policy θ under which the induced equilibrium allocation $\mathbf{A}^*(\tau^{\mathrm{eff}}, \theta)$ coincides with the optimal allocation $\mathbf{A}^{\mathrm{opt}}(\omega)$ defined in (49) in which each region attains the optimal consumption share $\mu^{\mathrm{opt}}(\omega)$. As before, let τ^{eff} be the efficient tax policy which implements the efficient aggregate allocation $\bar{\mathbf{A}}^{\mathrm{eff}}$ and, by Lemma 3 also determines the price system $\mathbf{P}^{\mathrm{eff}}$ supporting the efficient allocation. Let $W^{\ell,\mathrm{eff}}$ denote the induced lifetime non-transfer income of consumers in region ℓ defined in (31), $W^{\mathrm{eff}} := \sum_{\ell \in \mathbb{L}} W^{\ell,\mathrm{eff}}$ aggregate non-transfer lifetime income and T^{eff} the total tax revenue defined as in (15). Given a weighting scheme $\omega \in \Delta_+$ and consumption shares $\mu^{\mathrm{opt}}(\omega) = (\mu^{\ell,\mathrm{opt}}(\omega))_{\ell \in \mathbb{L}}$ determined by Lemma 4, consider the following transfer policy $\theta^{\mathrm{opt}} = (\theta^{\ell,\mathrm{opt}})_{\ell \in \mathbb{L}}$ defined for each $\ell \in \mathbb{L}$ as

$$\theta^{\ell,\mathrm{opt}}(\omega) = \frac{\mu^{\ell,\mathrm{opt}}(\omega)\left(W^{\mathrm{eff}} + T^{\mathrm{eff}}\right) - W^{\ell,\mathrm{eff}}}{T^{\mathrm{eff}}}. \tag{67}$$

Note that (67) determines consumer ℓ's lifetime cum-transfer income $W^{\ell} + T^{\ell}$ to be a share $\mu^{\ell,\mathrm{opt}}(\omega)$ of world cum-transfer income $W^{\mathrm{eff}} + T^{\mathrm{eff}}$. The following result shows that the transfer policy (67) together with τ^{eff} constitutes indeed an optimal climate policy.

Theorem 3
Let Assumptions 1 and 2 hold and define the climate tax policy τ^{eff} as in (64). Given any weighting scheme $\omega \in \Delta_+$, define $\mu^{\mathrm{opt}}(\omega)$ by (53) and the transfer policy $\theta^{\mathrm{opt}}(\omega)$ by (67). Then, the induced equilibrium allocation is ω-optimal, i.e., $\mathbf{A}^*(\tau^{\mathrm{eff}}, \theta^{\mathrm{opt}}(\omega)) = \mathbf{A}^{\mathrm{opt}}(\omega)$.

4.3 A Pareto-improving transfer policy

Applying the optimal transfer policy defined in (67) requires the choice of a particular weighting scheme $\omega \in \Delta_+$ or, equivalently, invoking Lemma 4 (ii), a desired consumption distribution $\mu = (\mu^\ell)_{\ell \in \mathbb{L}} \in \Delta_+$. This raises the question how such a distribution can and should be determined. Ideally, one might want to choose ω resp. μ as an equal weighting scheme based, e.g., on relative population sizes to ensure a fair world allocation of consumption. In any quantitative study of the model, however, such a choice would induce massive transfers unrelated to climate change but due to the fact that the world income distribution is very unequal and strongly biased towards industrialized countries.

The analysis of this paper, however, is not about fairness and redistribution of world income but how transfers can be determined such that each region has an incentive to implement the optimal tax on emissions. For this reason, the present section offers an alternative approach which chooses the consumption distribution based on the shares that each region attains in the Laissez faire allocation. This target seems a natural choice because the Laissez faire solution corresponds to the extreme case where all countries agree not to take any measures against climate change. It is therefore a natural threat point in any bargaining process about transfers.

Formally, let $\mu^{\mathrm{LF}} = (\mu^{\ell,\mathrm{LF}})_{\ell \in \mathbb{L}}$ denote the consumption shares along the Laissez faire equilibrium allocation $\mathbf{A}^{\mathrm{LF}}$ which are constant by Lemma 3 (ii). Using the same notation as in the previous subsection, define the transfer policy $\theta^{\mathrm{LF}} = (\theta^{\ell,\mathrm{LF}})_{\ell \in \mathbb{L}} \in \Delta$ as

$$\theta^{\ell,\mathrm{LF}} := \frac{\mu^{\ell,\mathrm{LF}} \left(W^{\mathrm{eff}} + T^{\mathrm{eff}} \right) - W^{\ell,\mathrm{eff}}}{T^{\mathrm{eff}}}, \quad \ell \in \mathbb{L}. \tag{68}$$

Under transfer policy θ^{LF}, each region ℓ attains the same relative wealth and the same share $\mu^{\ell,\mathrm{LF}}$ of world consumption along the efficient equilibrium allocation as in the Laissez faire allocation. The following main result shows that this policy makes each country better-off, i.e., consumers in each region enjoy utility strictly higher than in the Laissez faire allocation if they agree to jointly implement the efficient tax policy.[14]

Theorem 4
The equilibrium allocation $\mathbf{A}^*(\tau^{\mathrm{eff}}, \theta^{\mathrm{LF}})$ Pareto-improves the laissez faire allocation $\mathbf{A}^{\mathrm{LF}}$.

4.4 Emissions trading systems

Instead of levying a tax on emissions, an alternative policy to curb emissions is to introduce a global *emissions trading system (ETS)*. Such a system issues a certain number of emission allowances in each period which put an upper bound on Carbon emissions in that period. The possession of one emission allowance for period t allows the owner to emit one unit of fossil energy/carbon dioxide into the atmosphere in that period. For simplicity, assume that allowances are time-specific, i.e., can not be transferred across

[14]Clearly, this does not eliminate the free-riding problem that a single region may have an incentive to deviate from the optimal policy. A more elaborate game-theoretic analysis of this problem within the previous framework is beyond the scope of this paper but left for future research.

different periods. Then, the total number of allowances issued in period t puts an upper bound ('cap') on CO_2 emissions in that period. For this reason, an emissions-trading system is also called a 'cap-and trade system'. As the equilibrium effects of an emissions trading system are independent of the distribution of emission rights across sectors or regions, such a system can formally be defined a follows.

Definition 7
An Emissions Trading System (ETS) is a sequence $\{\bar{Z}_t\}_{t\geq 0}$ where $\bar{Z}_t \geq 0$ is the number of emission allowances issued in period t.

In each period t, a dirty sector $i \in \mathbb{I}_x$ in region ℓ needs to cover its emissions by purchasing an equal number of emission allowances from the ETS at the prevailing carbon price $p_t^Z \geq 0$.[15] The latter is determined endogenously. Thus, the demand of this sector for allowances in period t is $Z_{i,t}^\ell = \zeta_i X_{i,t}^\ell$. Market clearing on the Carbon market requires

$$\sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} Z_{i,t}^\ell \leq \bar{Z}_t. \tag{69}$$

Condition (69) holds with equality if $p_t^Z > 0$ in which case total emissions (5) coincide with the cap $\bar{Z}_t$ set by the ETS. Conversely, if $\sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} Z_{i,t}^\ell < \bar{Z}_t$, then $p_t^Z = 0$. The latter holds if production plans do not exhaust the total number of emission allowances which become worthless in this case. This holds, for example, if the number of allowances in the market exceeds the emissions along the Laissez faire equilibrium in each period.

One can now easily define an equilibrium where an ETS $\bar{Z} = (\bar{Z}_t)_{t\geq 0}$ replaces the tax system τ in Definition 3. In addition to the variables listed in Definition 3, the equilibrium determines a price sequence $(p_t^{Z*})_{t\geq 0}$ such that (69) holds for all $t \geq 0$. Here, we omit such a formal definition which is straightforward. Denote by $\mathbf{A}^*(\bar{Z}, \theta)$ the resulting equilibrium allocation with the transfer policy θ defined as before.

4.5 Optimal emissions trading systems

Defining the efficient aggregate allocation $\mathbf{A}^{\text{eff}}$ as in (52), let $(Z_t^{\text{eff}})_{t\geq 0}$ denote the induced global CO_2 emissions determined by (5). We now have the following result.

Theorem 5
Let Assumptions 1 and 2 hold. Then, the ETS $\bar{Z}^{\text{eff}} = (Z_t^{\text{eff}})_{t\geq 0}$ induces an aggregate equilibrium allocation which is efficient, i.e., $\bar{\mathbf{A}}^*(\bar{Z}^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$.

[15] The assumptions that dirty sectors must purchase all emission rights and do not receive them for free as initial endowments is without loss of generality. One can easily show that such an endowment enters the profit maximization problems as an additive constant and, therefore, has no effect on the behavior of the firm. The revenue received by consumers as a transfer from the ETS in the present case are replaced one-for one by profits from dirty sectors corresponding to the value of their endowment.

The previous result shows that the efficient allocation can be implemented by issuing emission allowances ('caps') equal to the optimal emissions path determined by the efficient allocation. Further, the equilibrium Carbon price sequence $(p_t^{Z*})_{t\geq 0}$ coincides with the optimal tax defined in (56). Hence, we can directly interpret this tax as the price for CO_2. Given a weighting scheme $\omega \in \Delta_+$, one can define the transfer policy as in (67) to obtain an equilibrium allocation $\mathbf{A}^*(\bar{Z}^{\text{eff}}, \theta^{\text{opt}})$ which is ω-optimal. Here, the transfer policy would distribute the revenue from auctioning the emission allowances in each period across countries.

5 Conclusions

The problem of determining an optimal climate path and an efficient allocation of production factors and exhaustible resources admits a unique solution which is completely independent of the interests of different countries. This solution can be implemented as a decentralized equilibrium by levying a uniform global tax on carbon emission which can be computed (or approximated) in closed form. Alternatively, this can be achieved by a globally organized emissions trading system where the 'caps' are uniquely determined by the efficient climate path. In principle, all countries should agree on this policy.

The real issue in the political debate about climate change is therefore not how and where emissions should be taxed, but rather how countries should share the tax revenue via transfers. These transfers determine the world distribution of consumption or income and provide a mechanism to compensate regions for climate damages. As the choice of an optimal transfer policy induces a trade-off between the interests of different countries, one might want to determine the transfer policy such that each region has an incentive to implement the optimal tax policy. An example of a transfer policy which leads to a Pareto-improvement relative to the Laissez faire allocation was devised in this paper.

The latter result is only a first step towards a more elaborate model of the political process which determines climate policies. In future research, we intend to model this process as a (cooperative or non-cooperative) game between regions as in Dutta & Radner (2006). Within the framework of this paper, $\mathbb{L}$ would be the set of players each of which chooses a domestic emissions tax policy τ^ℓ as their strategy and receives utility of domestic consumers as their pay-off. Transfers across regions then correspond to side payments which can be used to incentivize each region to implement a certain strategy. This raises the question whether there exists a transfer policy under which the optimal climate tax policy derived in this paper can be obtained as the Nash equilibrium of this non-cooperative game. In addition, one could also look a cooperative versions of this game where regions join forces to combat climate change by forming coalitions.

In addition, the framework developed in this paper can be extended in various directions. One such extensions is to replace the deterministic setup by a stochastic environment with random perturbations which allows to include various forms of uncertainty into the model. A second extension is a setup with endogenous growth and directed technical change as in Acemoglu, Aghion, Bursztyn & Hemous (2012). Both modifications were considered in GHKT and we believe that our framework is also amendable to them.

A Mathematical Appendix

A.1 Proof of Lemma 1

Using standard Lagrangian arguments, a non-negative sequence $(X_t^*)_{t\geq 0}$ is a solution to (23) if $\sum_{t=0}^{\infty} X_t^* \leq R_{i,0}^{\ell}$ and there exist non-negative Lagrangian variables $(\sigma_t^*)_{t\geq 0}$ and $\mu \geq 0$ such that $((X_t^*, \sigma_t^*)_{t\geq 0}, \mu)$ solve

$$q_t(v_{i,t} - c_i) + \sigma_t - \mu = 0 \qquad \forall t \geq 0 \tag{A.1a}$$

$$\sigma_t X_t = 0 \qquad \forall t \geq 0 \tag{A.1b}$$

$$\mu\Big(\sum_{t=0}^{\infty} X_t - R_{i,0}^{\ell}\Big) = 0. \tag{A.1c}$$

If $X_t^* > 0$ for all $t \geq 0$, then, $\sigma_t = 0$ by (A.1b) and $v_{i,t} \geq c_i$ by (A.1a) for all t. Using $q_0 = 1$ and $q_t/q_{t-1} = r_t^{-1}$ for all $t > 0$ in (A.1a), resource prices must evolve as in (24). The remaining assertions follow immediately. ■

A.2 Proof of Lemma 2

Under Assumption 2, (12) and (34) imply that the solution to (32) evolves as

$$C_t^{\ell *} = C_{t-1}^{\ell *}(\beta r_t)^{\frac{1}{\sigma}} = C_0^{\ell *}\prod_{s=1}^{t}(\beta r_s)^{\frac{1}{\sigma}} = C_0^{\ell *}\left(\frac{\beta^t}{q_t}\right)^{\frac{1}{\sigma}} \qquad t \geq 1. \tag{A.2}$$

Using (A.2), the l.h.s. in the lifetime budget constraint (30) can be written as

$$\sum_{t=0}^{\infty} q_t C_t^{\ell *} = \sum_{t=0}^{\infty} q_t C_0^{\ell *}\left(\beta^t/q_t\right)^{\frac{1}{\sigma}} = C_0^{\ell *}\sum_{t=0}^{\infty}\left(\beta^t q_t^{\sigma-1}\right)^{\frac{1}{\sigma}}. \tag{A.3}$$

Using (A.3) in (30) - which holds with equality- gives

$$C_0^{\ell *} = \frac{W^{\ell} + T^{\ell}}{\sum_{t=0}^{\infty}\left(\beta^t q_t^{\sigma-1}\right)^{\frac{1}{\sigma}}}. \tag{A.4}$$

Using (A.4) in (A.2) yields (36). ■

A.3 Proof of Lemma 3

(i) This is a direct consequence of the equilibrium conditions (18), (20), (22) and the boundary behavior of F_i and u imposed by Assumptions 1 and 2.

(ii) Set $\mu^{\ell *} := C_0^{\ell *}/\bar{C}_0^*$ for $\ell \in \mathbb{L}$. By Lemma 2, the growth rates of each sequence $(C_t^{\ell *})_{t\geq 0}$ are independent of ℓ and equal to the growth rates of aggregate consumption $(\bar{C}_t^*)_{t\geq 0}$. Induction then implies that (44) holds for all $t \geq 0$.

(iii) Let $i \in \mathbb{I}_x$ be arbitrary. If $R_{i,0} = \infty$, there exists a region $\ell \in \mathbb{L}$ for which $R_{i,0}^\ell = \infty$. As profits must be finite at equilibrium, (25) implies $v_{i,0} - c_i$.
If $R_{i,0} < \infty$, (41) implies $\lim_{t\to\infty} X_{i,t}^\ell = 0$ for all $\ell \in \mathbb{L}$. The boundary behavior from Assumption 1 gives $\lim_{t\to\infty} \partial_X F_i\big(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\big) = \infty$ and the claim therefore follows from (22c).
If $R_{i,0} > \sum_{t=0}^\infty \sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell *}$, the same arguments employed to prove (b) give the second result while the first one follows from Lemma 1 (ii). ■

A.4 Proof of Proposition 1

(i) Using the result from Lemma 2, aggregate consumption in period t satisfies

$$\bar{C}_t^* = \frac{(\beta^t/q_t)^{\frac{1}{\sigma}} \left[\sum_{\ell\in\mathbb{L}} W^\ell + T\right]}{\sum_{s=0}^\infty q_s \left(\beta^s/q_s\right)^{\frac{1}{\sigma}}} \qquad t \geq 0 \tag{A.5}$$

and determines K_{t+1} by (42). These and all other equations relevant to determine the variables in $\bar{\mathbf{A}}^*$ and prices $\mathbf{P}^*$ are independent of θ.

(ii) Using Lemma 2 and (14) in conjunction with (46) and (A.5) gives

$$C_t^{\ell *} = \frac{(\beta^t/q_t)^{\frac{1}{\sigma}} \left[W^\ell + \theta^\ell T\right]}{\sum_{s=0}^\infty q_s \left(\beta^s/q_s\right)^{\frac{1}{\sigma}}} = \mu^{\ell *}\bar{C}_t^*$$

for all $t \geq 0$ and $\ell \in \mathbb{L}$, proving the claim. ■

A.5 Proof of Lemma 4

(i) Let $\omega \in \Delta_+$ and $\bar{C} > 0$ be given. It is clear that $\omega^\ell = 0$ implies $\mu^\ell = 0$ which is implied by the solution (54). As we can always restate problem (54) as a maximization problem involving only those μ^ℓ for which $\omega^\ell > 0$, the remainder of the proof assumes w.l.o.g. that $\omega \gg 0$. The boundary behavior of u defined in (11) implies that any solution to (53) is bounded away from zero by, say, $\underline{\mu} = (\underline{\mu}^\ell)_{\ell\in\mathbb{L}} \gg 0$. This and the constraint $\sum_{\ell\in\mathbb{L}} \mu^\ell \leq 1$ define a compact, convex subset of $\mathbb{R}_{++}^L$ on which the map $\mu \mapsto \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell)$ is continuous and strictly concave ensuring that (53) has a unique solution μ^{opt}. Standard Lagrangian type arguments imply the existence of a multiplier $\lambda > 0$ such that the solution satisfies $\mu^\ell = (\omega^\ell \bar{C}^{1-\sigma}/\lambda)^{\frac{1}{\sigma}}$ for all $\ell \in \mathbb{L}$ and $\sum_{\ell\in\mathbb{L}} \mu^\ell = 1$. Combining these conditions to eliminate $\bar{C}^{1-\sigma}/\lambda$ gives (54). For later reference, let $a = 0$ if $\sigma \neq 1$ and $a = 1$ otherwise and use (11) to write the objective function in (53) as

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell \bar{C}) = a\left(u(\bar{C}) + \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell)\right) + (1-\sigma)\, u(\bar{C}) \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell). \tag{A.6}$$

Using (A.6), the maximum value (53) can be expressed as

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\mathrm{opt}} \bar{C}) = \begin{cases} (1-\sigma) m(\omega) u(\bar{C}) & \sigma \neq 1 \\ m(\omega) + u(\bar{C}) & \sigma = 1 \end{cases} \tag{A.7}$$

where $m(\omega) := \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\mathrm{opt}}(\omega))$. Note that $(1-\sigma)m(\omega) > 0$ whenever $\sigma \neq 1$.

(ii) Let $\tilde{\mu} = (\tilde{\mu}^\ell)_{\ell\in\mathbb{L}} \in \Delta_+$ be arbitrary. Defining $\tilde{\omega}^\ell := (\tilde{\mu}^\ell)^\sigma (\sum_{k\in\mathbb{L}} (\tilde{\mu}^k)^\sigma)^{-1}$ for each $\ell \in \mathbb{L}$, one verifies directly that $\tilde{\mu} = \mu^{\mathrm{opt}}(\tilde{\omega})$ solves (53) under this weighting scheme. ■

A.6 Proof of Theorem 1

Let a weighting scheme $\omega \in \Delta_+$ be arbitrary but fixed and $\mu^{\mathrm{opt}}(\omega)$ be the unique solution to (53). Denote the efficient solution (52) to the APP (51) by $\bar{\mathbf{A}}^{\mathrm{eff}} = (\bar{C}_t^{\mathrm{eff}}, \Gamma_t^{\mathrm{eff}})_{t\geq 0} \in \bar{\mathbb{A}}$ and define $\mathbf{A} = (\mathbf{C}_t, \Gamma_t^{\mathrm{eff}})_{t\geq 0} \in \mathbb{A}$ where $\mathbf{C}_t = (C_t^\ell)_{\ell\in\mathbb{L}} := \mu^{\mathrm{opt}}(\omega)\bar{C}_t^{\mathrm{eff}}$ as in the theorem. To establish that $\mathbf{A}$ is ω-optimal, i.e., solves (48), let $\mathbf{A}' = (\mathbf{C}'_t, \Gamma'_t)_{t\geq 0} \in \mathbb{A}$ be any other feasible allocation where $\mathbf{C}'_t = (C_t^{\ell\prime})_{\ell\in\mathbb{L}}$, $t \geq 0$. We have to show that

$$V((\mathbf{C}'_t)_{t\geq 0}; \omega) \leq V((\mathbf{C}_t)_{t\geq 0}; \omega). \tag{A.8}$$

Define aggregate consumption $(\bar{C}'_t)_{t\geq 0}$ induced by $(\mathbf{C'}_t)_{t\geq 0}$ as $\bar{C}'_t := \sum_{\ell\in\mathbb{L}} C_t^{\ell\prime}$, $t \geq 0$. Then, $(\bar{C}'_t, \Gamma'_t)_{t\geq 0} \in \bar{\mathbb{A}}$ and, since $(\bar{C}_t^{\mathrm{eff}}, \Gamma_t^{\mathrm{eff}})_{t\geq 0}$ solves the APP (51),

$$\sum_{t=0}^{\infty} \beta^t u(\bar{C}'_t) \leq \sum_{t=0}^{\infty} \beta^t u(\bar{C}_t^{\mathrm{eff}}). \tag{A.9}$$

Let $a = 1$ if $\sigma = 1$ and $a = 0$ otherwise. By (53) and (A.7), we have for all $t \geq 0$

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(C_t^{\ell\prime}) \leq \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\mathrm{opt}}(\omega)\bar{C}'_t) = a\left(m(\omega) + u(\bar{C}'_t)\right) + (1-\sigma)m(\omega)u(\bar{C}'_t) \tag{A.10}$$

and

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(C_t^\ell) = \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\mathrm{opt}}(\omega)\bar{C}_t^{\mathrm{eff}}) = a\Big(m(\omega) + u(\bar{C}_t^{\mathrm{eff}})\Big) + (1-\sigma)m(\omega)u(\bar{C}_t^{\mathrm{eff}}). \tag{A.11}$$

Equations (A.10) and (A.11) being true for all $t \geq 0$ and (A.9) then give

$$\begin{aligned}
V((\mathbf{C'}_t)_{t\geq 0};\omega) &= \sum_{t=0}^{\infty}\beta^t\sum_{\ell\in\mathbb{L}}\omega^\ell u(C_t^{\ell\prime}) \\
&\leq a\left(\frac{m(\omega)}{1-\beta}+\sum_{t=0}^{\infty}\beta^t u(\bar{C}'_t)\right)+(1-\sigma)m(\omega)\sum_{t=0}^{\infty}\beta^t u(\bar{C}'_t) \\
&\leq a\left(\frac{m(\omega)}{1-\beta}+\sum_{t=0}^{\infty}\beta^t u(\bar{C}_t^{\text{eff}})\right)+(1-\sigma)m(\omega)\sum_{t=0}^{\infty}\beta^t u(\bar{C}_t^{\text{eff}}) \\
&= \sum_{t=0}^{\infty}\beta^t\sum_{\ell\in\mathbb{L}}\omega^\ell u(C_t^\ell) = V((\mathbf{C}_t)_{t\geq 0};\omega)
\end{aligned}$$

This proves (A.8) and the claim. ■

A.7 Computing the efficient solution (52)

We adopt a standard Langrangian approach also used in GHKT to characterize the solution (52). For brevity, use (43) setting $\mathbf{E}_t^\ell := (E_{i,t}^\ell)_{i\in\mathbb{I}}$, $D_t^\ell := D^\ell(S_{1,t}+S_{2,t})$. Define Lagrangen multipliers $\lambda_t := (\lambda_{0,t}, ((\lambda_{i,t}^\ell)_{i\in\mathbb{I}_0}, \lambda_{N,t}^\ell)_{\ell\in\mathbb{L}}, \lambda_{K,t}, \lambda_{S_1,t}, \lambda_{S_2,t})$, $\lambda = (\lambda_t)_{t\geq 0}$ and $\mu = (\mu_i)_{i\in\mathbb{I}_x}$ and the Lagrangian function

$$\begin{aligned}
&\mathscr{L}\Big(\big(\bar{C}_t, K_{t+1}, \mathbf{K}_t, \mathbf{N}_t, \mathbf{E}_t, \mathbf{X}_t, \mathbf{S}_t\big)_{t\geq 0}, \lambda, \mu\Big) := \sum_{t=0}^{\infty}\beta^t\Bigg[u(\bar{C}_t) \\
&+ \lambda_{0,t}\Bigg(\sum_{\ell\in\mathbb{L}}\big(1-D^\ell(S_{1,t}+S_{2,t})\big)Q_{0,t}^\ell F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) - C_t - K_{t+1} - \sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x} c_i X_{i,t}^\ell\Bigg) \\
&+ \sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}\lambda_{i,t}^\ell\Big(Q_{i,t}^\ell F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - E_{i,t}^\ell\Big) + \sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}\setminus\mathbb{I}_x}\lambda_{i,t}^\ell\Big(Q_{i,t}^\ell F_i(K_{i,t}^\ell, N_{i,t}^\ell) - E_{i,t}^\ell\Big) \\
&+ \sum_{\ell\in\mathbb{L}}\lambda_{N,t}^\ell\Big(N_t^{\ell,s} - \sum_{i\in\mathbb{I}_0}N_{i,t}^\ell\Big) + \lambda_{K,t}\Big(K_t - \sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_0}K_{i,t}^\ell\Big) \\
&+ \lambda_{S_1,t}\Bigg(S_{1,t} - S_{1,t-1} - \phi_L\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}\zeta_i X_{i,t}^\ell\Bigg) \\
&+ \lambda_{S_2,t}\Bigg(S_{2,t} - (1-\phi)S_{2,t-1} - (1-\phi_L)\phi_0\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}\zeta_i X_{i,t}^\ell\Bigg)\Bigg] \\
&+ \sum_{i\in\mathbb{I}_x}\mu_i\left(R_{i,0} - \sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}X_{i,t}^\ell\right).
\end{aligned}$$

Standard arguments imply that any solution $A = (\bar{C}_t, K_{t+1}, \mathbf{K}_t, \mathbf{N}_t, \mathbf{E}_t, \mathbf{X}_t, \mathbf{S}_t)_{t\geq 0}$ to (51) has to satisfy the first order and complementary slackness conditions. After eliminating as many Lagrangian variables as possible, these conditions can be summarized for all $\ell \in \mathbb{L}$, $i \in \mathbb{I}$, and $t \geq 0$ (suppressing quantifiers when convenient) as:

$$
\begin{aligned}
u'(\bar{C}_t) &= \lambda_{0,t} = \beta\lambda_{K,t+1} && \text{(A.12a)}\\
\lambda_{K,t} &= \lambda_{0,t}(1-D_t^\ell)Q_{0,t}^\ell \partial_K F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) && \text{(A.12b)}\\
\lambda_{N,t}^\ell &= \lambda_{0,t}(1-D_t^\ell)Q_{0,t}^\ell \partial_N F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) && \text{(A.12c)}\\
\lambda_{i,t}^\ell &= \lambda_{0,t}(1-D_t^\ell)Q_{0,t}^\ell \partial_{E_i} F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) && \text{(A.12d)}\\
\mu_i &= \beta^t\Big[\lambda_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \lambda_{0,t}c_i - \lambda_{0,t}\zeta_i\Lambda_t\Big] && \text{(A.12e)}\\
\lambda_{S_1,t} &= \lambda_{0,t}\sum_{\ell\in\mathbb{L}} \frac{dD^\ell(S_t)}{dS}\frac{Y_t^\ell}{1-D^\ell(S_t)} + \beta\lambda_{S_1,t+1} && \text{(A.12f)}\\
\lambda_{S_2,t} &= \lambda_{0,t}\sum_{\ell\in\mathbb{L}} \frac{dD^\ell(S_t)}{dS}\frac{Y_t^\ell}{1-D^\ell(S_t)} + (1-\phi)\beta\lambda_{S_2,t+1}. && \text{(A.12g)}
\end{aligned}
$$

Note that $\lambda_{0,t}$ can be interpreted as a shadow price of time t consumption. Thus, the time t shadow price of energy of type $i \in \mathbb{I}$ produced in region $\ell \in \mathbb{L}$ measured in time t consumption goods can be defined as

$$\hat{p}_{i,t}^\ell := \frac{\lambda_{i,t}^\ell}{\lambda_{0,t}}. \tag{A.13}$$

Combining (A.12a) and (A.12b) gives the familiar Euler equation

$$u'(\bar{C}_{t-1}) = \beta u'(\bar{C}_t)(1-D^\ell(S_t))Q_{0,t}^\ell \partial_K F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}\big). \tag{A.14}$$

As the l.h.s in (A.12e) is independent of ℓ and t, we obtain using (A.13)

$$\hat{p}_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \zeta_i\Lambda_t = \hat{p}_{i,t}^{\ell'} Q_{i,t}^{\ell'} \partial_X F_i(K_{i,t}^{\ell'}, N_{i,t}^{\ell'}, X_{i,t}^{\ell'}) - \zeta_i\Lambda_t =: \hat{v}_{i,t} \tag{A.15}$$

for all $\ell, \ell' \in \mathbb{L}$ and $t \geq 0$ and

$$\hat{v}_{i,t} - c_i = \frac{\beta u'(\bar{C}_{t+1})}{u'(\bar{C}_t)}\Big(\hat{v}_{i,t+1} - c_i\Big) \tag{A.16}$$

for all $t \geq 0$. Essentially, (A.15) ensures intratemporal and (A.16) intertemporaly efficiency of resource extraction. Analogously to the proof of Lemma 3 (iii), one can easily show that (41) in conjunction with Assumption 1 exclude a solution $\mu_i = 0$. Thus, $\mu_i > 0$ and the resource constraint (41) is always binding.

Assuming that $\lim_{n\to\infty}\beta^{n+1}\lambda_{S_1,t+n} = \lim_{n\to\infty}((1-\phi)\beta)^{n+1}\lambda_{S_2,t+n} = 0$, (A.12f) and (A.12g) can be solved forward to obtain a result similar to GHKT:

$$
\begin{aligned}
\frac{\lambda_{S_1,t}}{\lambda_{0,t}} &= \sum_{n=0}^{\infty}\beta^n\frac{u'(C_{t+n})}{u'(C_t)}\sum_{\ell\in\mathbb{L}}\frac{dD^\ell(S_{t+n})}{dS}\frac{Y_{t+n}^\ell}{1-D^\ell(S_{t+n})} && \text{(A.17a)}\\
\frac{\lambda_{S_2,t}}{\lambda_{0,t}} &= \sum_{n=0}^{\infty}\beta^n\frac{u'(C_{t+n})}{u'(C_t)}(1-\phi)^n\sum_{\ell\in\mathbb{L}}\frac{dD^\ell(S_{t+n})}{dS}\frac{Y_{t+n}^\ell}{1-D^\ell(S_{t+n})}. && \text{(A.17b)}
\end{aligned}
$$

Finally, define for $t \geq 0$

$$\Lambda_t := \phi_L \frac{\lambda_{S_1,t}}{\lambda_{0,t}} + (1-\phi_L)\phi_0 \frac{\lambda_{S_2,t}}{\lambda_{0,t}}. \tag{A.18}$$

Then, using (A.17a) and (A.17b) in (A.18) gives precisely the condition (56).

A.8 Proof of Theorem 2

Combining (64) with the conditions derived in Section 2, one can show directly that the aggregate equilibrium allocation solves the same equations as the efficient solution which were derived in Section 3.5 resp. A.7. This proves the claim. ■

A.9 Proof of Theorem 3

The assertion follows directly from Proposition 1 and the transfer policy (67). ■

A.10 Proof of Theorem 4

Let $\ell \in \mathbb{L}$ be arbitrary and $(\bar{C}_t^{\text{eff}})_{t\geq 0}$ and $(\bar{C}_t^{LF})_{t\geq 0}$ be the aggregate consumption sequences along the efficient and laissez faire allocation, respectively. By Lemma 3(ii) and Assumption 2, utility of region ℓ along the LF allocation is $U((\mu_\ell^{\text{LF}}\bar{C}_t^{LF})_{t\geq 0}) = a + b\,U((\bar{C}_t^{LF})_{t\geq 0})$ where a and $b > 0$ are constants that depend only on μ^{LF}. Further, by construction and Lemma 3(ii), utility of region ℓ along the allocation $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{LF}})$ is $U((\mu_\ell^{\text{LF}}\bar{C}_t^{\text{eff}})_{t\geq 0}) = a + b\,U((\bar{C}_t^{\text{eff}})_{t\geq 0})$. Thus, $U((\mu_\ell^{\text{LF}}\bar{C}_t^{LF})_{t\geq 0}) < U((\mu_\ell^{\text{LF}}\bar{C}_t^{\text{eff}})_{t\geq 0})$ if and only if $U((\bar{C}_t^{LF})_{t\geq 0}) < U((\bar{C}_t^{\text{eff}})_{t\geq 0})$ which follows directly from the optimality of the efficient allocation (52). ■

A.11 Proof of Theorem 5

For each $t \geq 0$, set the Carbon price p_t^{Z*} equal to the optimal tax defined in (56). Then, analogously to Theorem 2 , the aggregate equilibrium allocation solves the same equations as the efficient solution which were derived in Section 3.5 resp. A.7. ■

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