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A Survey of Location-Routing Problems

Michael Drexl and Michael Schneider

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Johannes Gutenberg University Mainz
Gutenberg School of Management and Economics
Jakob-Welder-Weg 9
55128 Mainz
Germany
wiwi.uni-mainz.de

Contact details

Michael Drexl

Department Optimization

Fraunhofer Centre for Applied Research on Supply Chain Services SCS Nordostpark 93

90411 Nuremberg

michael.drexl@scs.fraunhofer.de

Michael Schneider

Logistics Planning and Information Systems DB Schenker Endowed Assistant Professorship Department of Law and Economics, TU Darmstadt Hochschulstraße 1

64289 Darmstadt

Germany

schneider@bwl.tu-darmstadt.de

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Michael Drexl
Chair of Logistics Management, Gutenberg School of Management and Economics,
Johannes Gutenberg University, Mainz
and
Fraunhofer Centre for Applied Research on Supply Chain Services SCS, Nuremberg
E-mail: michael.drexl@scs.fraunhofer.de

Michael Schneider
Logistics Planning and Information Systems
DB Schenker Endowed Assistant Professorship
Department of Law and Economics, TU Darmstadt
E-mail: schneider@bwl.tu-darmstadt.de

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Abstract

This paper gives a review of the recent literature on location-routing problems published since the last survey of Nagy and Salhi appeared in 2006. We propose a classification of problem variants, provide concise paper excerpts that convey the central ideas of each work, discuss recent developments in the field, and list promising topics for further research.
Keywords: Survey; Location-routing.

1 Introduction

Location-Routing Problems (LRPs) combine two basic planning tasks in logistics. In LRPs, as their name implies, decisions on the location of arbitrary types of facilities (plants, depots, warehouses, hubs, cross-docks etc.) are jointly taken with decisions on the routing of vehicles. It is well-known that making these types of decisions independently of one another may lead to highly suboptimal planning results (Salhi and Rand 1989), even if the location decisions must be made for the long term (Salhi and Nagy 1999). Consequently, LRPs have been studied for decades, and the research community has been very active, particularly in the last years.
This paper provides an overview of the recent literature on LRPs. Earlier surveys were published by Balakrishnan et al. (1987), Laporte (1988), Laporte (1989), Berman et al. (1995), Min et al. (1998), and Nagy and Salhi (2007). We considered it worthwhile to compile a new literature review for several reasons: The most recent survey, by Nagy and Salhi, was made available online more than seven years ago. Since then, numerous papers have appeared, and significant new developments can be observed. Notably, two trends shall be pointed out. First, in contrast to the situation described by Nagy and Salhi, who remarked that LRP research is rather fragmented and that no widely accepted benchmarks exist, recent years have witnessed a new approach to LRPs. Often the same or similar problems are addressed using standardized benchmarks, thus allowing a comparison of algorithms. Second, there is a rather general trend in logistics planning to 'richer', more comprehensive and integrated models (Hartl et al. 2006, Drexl 2012), and this is reflected in the newer literature on LRPs (see Section 11 below).

Specifically, in this paper, we subsume under the term *location-routing problem* a mathematical optimization problem where at least the following two types of decisions must be made interdependently:

(i) Which facilities out of a finite or infinite set of potential ones should be used (for a certain purpose)?
(ii) Which vehicle routes should be built, i.e., which customer clusters should be formed and in which sequence should the customers in each cluster be visited by a vehicle from a given fleet (to perform a certain service)?

In other words, we are interested in problems where, on the one hand, facilities must be selected, but where this selection is not implicitly determined by the routing decisions, and where, on the other hand, routes for vehicles must be determined, not only assignments of customers or flows of goods. In particular, the selection of facilities will not be implicitly determined by the routing decisions if

(i) there are fixed costs for opening and/or variable (volume-dependent) costs for using a facility or
(ii) (exactly or at most) a given number of facilities must be selected out of a larger set or
(iii) the facilities have some kind of capacity limitation.

We use these criteria as a general guideline for limiting the material discussed in this review. Thus, problems not studied here are pure facility location problems (FLPs, Daskin 1995), and (service) network design problems (Crainic and Kim 2007, Wieberneit 2008). In these problems, no vehicle routing is performed. Moreover, we exclude vehicle routing-allocation/median cycle problems (Nagy and Salhi 2007), as for these problems, none of the three items listed in the previous paragraph applies. Hamiltonian p-median problems (Branco and Coelho 1990) are also omitted; we do not consider this problem type an LRP because it actually requires no locational decision. Finally, we do not cover the following problems because in all of them, the location decisions are implicitly determined by the routing: multi-depot vehicle routing problems (MDVRPs, Cordeau et al. 1997), VRPs with intermediate depots or refill points (Ghiani et al. 2001, Tarantilis et al. 2008), pickup-and-delivery problems with transshipments, vehicle and driver routing and scheduling problems with driver changes at relay stations en route, and VRPs with trailers and transshipments (see Drexl 2012 for a survey of the last three problems).

Similar to many review articles, this paper is not intended to cover the complete LRP literature, reviewing and categorizing papers that have possibly been reviewed even more than once in the past. Instead, we provide a literature update and restrict ourselves to papers that (i) were published between 2006 and 2013, and (2) were not already discussed in the survey of Nagy and Salhi (2007).

We have included journal articles, conference proceedings, technical reports, and Ph.D. dissertations written in English. We do not claim to have collected all the LRP literature from 2006 onwards, but we think we have identified a representative subset of the work carried out by the research community since then.

Our aim is to provide sufficiently detailed excerpts so that the central ideas and unique features of each work become clear. Excerpt lengths vary and depend on several factors such as the complexity of the conveyed ideas, the length of the original paper, the similarity of the original paper to previously discussed works and concepts, and, to some extent also on our own subjective opinion on the importance of an article. With respect to terminology, we use the term *facility* throughout the paper to denote the objects to be located, no matter what these objects are in an actual or potential application context. Thus, saying 'a facility must be located' either means that a location must be determined where a facility with certain capabilities is to be installed, or that an existing facility (or a part of it) must be used, rented, or bought to support operations. It is assumed that the reader is familiar with the basics of location theory, vehicle routing, and exact and heuristic solution techniques for combinatorial optimization problems in general. Pertinent textbooks are, respectively, Daskin (1995), Toth and Vigo (2002) or Golden et al. (2008), Wolsey (1998), and Gendreau and Potvin (2010).

The rest of the paper is structured as follows. In Section 2, different types and characteristics of LRPs are described. Section 3 describes widely used sets of benchmark instances. Sections 4–11 review literature on different variants of LRPs. In these sections, excerpts of relevant papers are presented. Papers are mostly listed in chronological order, but closely related works by the same researchers are sometimes described together. Section 12 summarizes the key insights we gained during our study, concerning problems, models, applications, and algorithms. To conclude, Section 13 suggests promising topics for further research. All abbreviations and acronyms used in the text are listed in the Appendix.

2 LRP variants

There are numerous types or variants of LRPs. In this section, we identify the most important criteria for categorizing the existing literature by problem type. The subsequent review sections are structured based on these criteria. We differentiate between so-called main characteristics that fundamentally change the nature of the problem, thus defining a new problem variant, and so-called subcharacteristics, for which this is not the case.

2.1 Main characteristics defining new problem variants

Deterministic vs. stochastic vs. fuzzy data. In a deterministic planning situation, all problem data are known in advance. Stochastic data means that some information (in most cases, customer demands or travel times) is given in the form of probability distributions. Fuzzy data means that some problem parameters are available in the form of fuzzy numbers. Most papers in this review assume deterministic data. We found four papers dealing with stochastic and four concerned with fuzzy data, see Section 8.

Discrete vs. continuous vs. network locations. In discrete problems, the potential locations for opening facilities are given as a (sub)set of vertices of a graph. In continuous or planar problems, the choice of facility locations is not restricted to a discrete set, but facilities may be located freely in the plane. In network location problems, a facility may be opened at any vertex of a graph/network or anywhere on a link (edge, arc). The large majority of papers considers discrete problems. The planar problem was first studied by Schwardt and Dethloff (2005) for locating a single facility. To the best of our knowledge, the only work dealing with the case of multiple facilities is (Salhi and Nagy 2009), which is already included in the review of Nagy and Salhi (2007). Other than that, we found only two papers on planar location, which are reviewed in Section 11. We found no paper considering network location.

Single vs. multiple echelons. The basic idea of Multi- or N-Echelon VRPs and LRPs (NE-VRPs/LRPs) is that customers are not served directly from a central depot but via N legs in an N-stage distribution network. An N-stage distribution network contains $N+1$ levels of locations. Echelon $n \in \{1, \ldots, N\}$ considers transports from location level $n-1$ to n, see Figure 1. For each echelon n, there are dedicated vehicles that can only visit the facilities defining echelon n. Load transfers are required between vehicles of different echelons. As can be seen in Section 5, many papers on multi-echelon LRPs have appeared in the last few years.

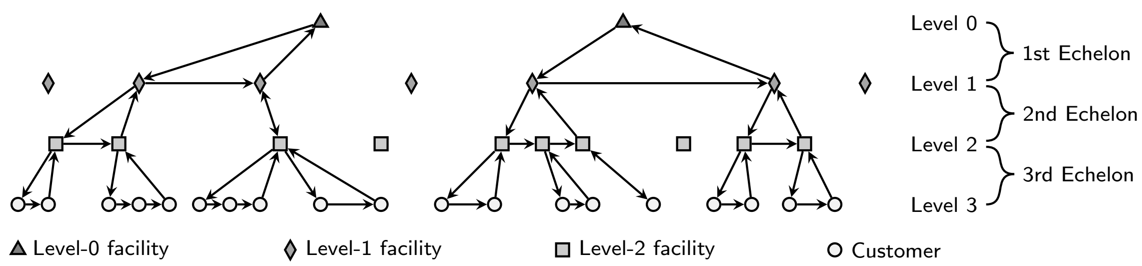

Figure 1: Example of a three-echelon routing problem

Static vs. dynamic vs. periodic problems. We use the term static to denote problems considering one single planning period. The term dynamic refers to problems with multiple planning periods where some information (usually customer demands) is initially unknown and becomes available over time. Periodic LRPs (PLRPs) comprise multiple planning periods and assume complete information on all relevant data. The aim of periodic problems is to determine visiting patterns for customers, i.e., to decide on the periods in which to visit each customer (see Section 6).
Single vs. multiple objective. Most papers consider a single objective such as minimization of the sum of fixed facility location costs and fixed and variable vehicle routing costs. Some works, though, deal with several objective functions simultaneously. Mostly, qualitative measures such as service levels are considered along with monetary objectives (see Section 9).
Inventory decisions. A natural extension of a problem containing facility location aspects is the consideration of inventory management at the facilities, i.e., how much of a good to keep in stock and when and how much to order from the manufacturer. Several papers have integrated such a component into an LRP (see Section 10).
Pickup-and-delivery LRPs. The tasks to be performed in LRPs may consist in delivering goods to customers from one of several potential facilities, in picking up goods at customers and delivering these goods to one of several potential facilities, or both. In this last case, it is possible that goods must be picked up at one customer and delivered to another. Such problems are called pickup-and-delivery LRPs. It is also possible that a single customer requires both a pickup and a delivery of goods, and that pickup and delivery at a customer have to be done during the same visit. This is called simultaneous delivery and pickup. Many-to-many LRPs are pickup-and-delivery problems where the planning goal is to locate a network of intermediate facilities or hubs for the transshipment of goods. Pickups and deliveries are performed on local, multi-stop routes starting and ending at a hub; inter-hub transports are usually direct. Such problems arise, e.g., in postal or parcel delivery applications. Papers on pickup-and-delivery and many-to-many LRPs are reviewed in Section 7.
Generalized LRPs (GLRPs). Similar to the well-known generalized traveling salesman problem (TSP) (Fischetti et al. 2002), in GLRPs, the customers are clustered into disjoint groups. The requirement in the GLRP is to find routes, starting and ending at a facility, so that exactly one customer from each group is visited exactly once. Obviously, the LRP variants discussed above can be regarded as GLRPs of the type where each group contains exactly one customer. We found only two papers on GLRPs (Glicksman and Penn 2008, Harks et al. 2013). These are reviewed in Section 11.
Prize-Collecting LRPs (PCLRPs). PCLRPs allow that some customers are not visited by any tour. For these customers, a per-customer penalty (e.g., an outsourcing cost) is incurred. The sum of fixed facility opening, variable tour and outsourcing costs is to be minimized. Also for PCLRPs, we found only two papers (Ahn et al. 2012, Harks et al. 2013), and these are reviewed in Section 11, too.
Split delivery LRPs (SDLRPs). The option of split delivery allows that a customer can be visited more than once and by more than one vehicle in order to fulfill his demand. There is quite a number of papers on VRPs with split delivery (see the survey by Archetti and Speranza 2008), whereas we have found only one paper (Harks et al. 2013) considering split delivery LRPs.
Location Arc-Routing Problems (LARPs). LARPs are LRPs where the required services must be performed along the links of a network instead of at vertices. Ghiani and Laporte (2001) present a survey on the topic. We have found only one paper (Hashemi Doulabi and Seifi 2013, see Section 11) that has appeared since then.

2.2 Subcharacteristics not defining new variants

Besides the above characteristics, LRPs may differ with respect to the following aspects:
- (Un)directed network
- (Un)capacitated facilities

- (No) fixed costs for opening a facility
- (Un)limited/(un)capacitated fleet
- Homo-/heterogeneous fleet

In our opinion, these aspects do not change the nature of a problem so much as to constitute a new LRP variant. The majority of the reviewed papers consider capacitated facilities with fixed costs for opening and with capacitated vehicles. Therefore, in the review sections, we only indicate exceptions from this 'rule'.

In this paper, the *standard* LRP is defined as a deterministic, static, discrete, single-echelon, single-objective problem where each customer must be visited exactly once. This standard LRP reduces to the TSP if there is only one potential facility and the fleet consists of only one uncapacitated vehicle. As the TSP is well-known to be NP-hard (Garey and Johnson 1979), so are the standard LRP and most of its variants and extensions.

3 Benchmark instances

We have pointed out in the Introduction that the recent literature has made extensive use of standardized benchmark instances to evaluate the quality of the developed solution procedures by comparing their performance with that of other algorithms. We consider this a valuable development. There are several sets of benchmark instances used in the surveyed literature. We list them by problem variant and in chronological order in Table 1. The following sections discuss the problem variants introduced above, starting with the standard LRP.

4 Works on the standard LRP

The majority of the LRP literature reviewed in the present paper falls into this category. To structure the pertinent material, the following subsections respectively discuss exact solution approaches, matheuristics, and classical metaheuristics.

4.1 Exact approaches

Berger et al. (2007) solve the standard LRP with uncapacitated facilities and vehicles, but with a bound on the maximal length of each route, by branch-and-price. Due to the route length constraint, more than one route may start and end at a facility. In addition to path variables for each feasible route starting and ending at the same facility, the authors introduce binary variables indicating whether or not a certain facility is opened in order to account for the fixed opening costs. The subproblem for generating new columns/paths is an elementary shortest path problem with one resource constraint (for the route length). It is solved by the labeling algorithm introduced in Feillet et al. (2004). Branching is first performed on the location variables, and then, for the path variables, a modification of the strategy proposed by Ryan and Foster (1981) is used. Computational experiments are performed with self-generated random instances with 10 potential facilities and between 50 and 100 customers. Several 100-customer instances are solved to optimality within a run-time of two hours.
Akca et al. (2009) describe three exact and one heuristic branch-and-price approach. The solution approaches differ with respect to how the pricing problem, an Elementary Shortest Path Problem with Resource Constraints (ESPPRC), is solved. In each approach, combinations of exact and heuristic variants of labeling algorithms for the ESPPRC are used. Furthermore, the authors describe five classes of valid inequalities. The number of these cuts is polynomial in the instance size, so they are directly added to the relaxed master problem to strengthen it, and no dynamic constraint generation is performed. Computational experiments are performed with selected Perl and B instances (those with less than 150 customers) and ABR instances (the latter are introduced in this paper). The largest instances solved to optimality have five facilities and 40 customers. Good upper bounds can be quickly determined. However, for several instances, the

Acronym	First reference; Remarks	No. instances	Size (min–max no. facilities/ min–max no. customers)	Link
LRP				
Perl	Introduced in the unpublished Ph.D. dissertation of Perl (1983).	5	2–15/12–318	sweet.ua.pt/sbarreto/_private/-SergioBarretoHomePage.htm
TB	Tuzun and Burke (1999)	36	10–20/100–200	prodhonc.free.fr/homepage
B	Introduced in the unpublished Ph.D. dissertation of Barreto (2003). Used in Barreto et al. (2007).	13	5–14/21–150	sweet.ua.pt/sbarreto/_private/-SergioBarretoHomePage.htm
ADF	Albareda-Sambola, Díaz, and Fernández (2005)	15	5–10/10–30	Not on the Internet
PPW	Prins, Prodhon, and Wolfler Calvo (2006a)	30	5–10/20–200	prodhonc.free.fr/homepage
ABR	Akca, Berger, and Ralphs (2009)	12	5/30–40	claudio.contardo.org/instances
BMW	Baldacci, Mingozzi, and Wolfler Calvo (2011)	4	14/150–199	claudio.contardo.org/instances
HKM	Harks, König, and Matuschke (2013)	27	100–1,000/ 1,000–10,000	www.coga.tu-berlin.de/v-menue/-download_media/clrlib
2E-LRP				
GPTV	Gonzalez Feliu, Perboli, Tadei, and Vigo (2008)	105	1/2–4/12–50	people.brunel.ac.uk/~mastjjb/-jeb/orlib/vrp2einfo.html
CPMT	Crainic, Perboli, Mancini, and Tadei (2010b). Instances with 50, 100, 150, and 250 customers. Strictly speaking, these are no LRP instances according to our criteria specified above because the selection of the potential facilities is unrestricted and implicitly determined by the routing decisions. The instances are, however, used in papers on 2E-LRPs.	132	1/2–10/50–250	people.brunel.ac.uk/~mastjjb/-jeb/orlib/vrp2einfo.html (only the instances with 50 customers)
NPP-N	Nguyen, Prins, and Prodhon (2010)	24	1/5–10/20–200	prodhonc.free.fr/homepage
NPP-P	Nguyen, Prins, and Prodhon (2010). PPW instances modified by adding one level-0 facility.	30	1/5–10/20–200	prodhonc.free.fr/homepage
S	Sterle (2010)	156	2–5/3–20/8–200	Not on the Internet
Others				
Prodhon	PLRP, Prodhon (2008)	30	5–10/20–200	prodhonc.free.fr/homepage
KAKD	LRPSPD, Karaoglan, Altiparmak, Kara, and Dengiz (2011). Modified B and PPW instances (those with up to 100 customers).	37	2–10/8–100	Not on the Internet

Table 1: Benchmark instances

algorithms have difficulties in raising the lower bounds in the course of the branch-and-bound process. The authors attribute this to the lack of dynamic cut generation.

Belenguer et al. (2011) present a branch-and-cut algorithm. They use a two-index formulation with two types of binary routing variables, the first one indicating whether or not an edge is traversed and the second one indicating whether an edge between a facility and a customer is traversed exactly twice (meaning a single-customer route). In addition, a binary variable for each potential facility indicates whether or not the facility is open. The authors introduce a number of valid inequalities to strengthen the LP relaxation, some of them derived from Capacitated VRP (CVRP) cuts, some of them lifted versions of constraints that are already present in the original Mixed-Integer Programming (MIP) formulation. Moreover, so-called optimality cuts are used, i.e., cuts that are not valid for all feasible integer solutions but only for at least one optimal solution. Exact and heuristic separation procedures are developed. Computational experiments are performed with the Perl, B, ABR, and PPW benchmark instances. For the ABR instances, the algorithm outperforms the one described in Akca et al. (2009). The largest instances solved to optimality have five facilities and 50 customers.

Baldacci et al. (2011) present an exact algorithm based on the formulation by Akca et al. (2009). The core observation used in the algorithm is the following: An optimal solution to the LRP can be computed when minimizing, over all subsets of the set of potential facilities, the opening costs of the facilities in a subset and the costs of an optimal solution to an MDVRP where the depots correspond to the facilities in the subset and have the respective capacities (MCDVRP). The method exploits the following three facts: (i) The number of facility subsets to be considered can be drastically reduced by eliminating subsets that cannot lead to an optimal solution. (ii) MCDVRPs are much easier to solve than LRPs. (iii) A good lower bound for the MCDVRP can be obtained by extending a previously published method (Baldacci and Mingozzi 2009) for the heterogeneous fleet VRP.

The algorithm consists of three stages and works as follows. First, a lower bound for the LRP is computed using two relaxations of the path-variable formulation of the LRP: a Single-Source Capacitated Facility Location Problem (SSCFLP) and a Multiple-Choice Knapsack Problem. Cost coefficients are appropriately set to reflect marginal routing costs for servicing a customer from a facility. Second, by means of a branch-and-bound algorithm, all relevant facility subsets are determined using a lower bound for the MCDVRP. This bound, in turn, is computed using results obtained during the solution of the two LRP relaxations and a known upper bound on the LRP. Third, the LRP is solved by iteratively selecting one of the relevant subsets that minimizes the sum of the opening costs of the facilities in the subset and the lower bound on the optimal solution value for the corresponding MCDVRP. In each iteration, for the currently selected subset, the MCDVRP is solved exactly by (i) computing lower bounds with the bounding procedures for the two LRP relaxations on the reduced LRP containing only the potential facilities in the current subset, (ii) computing a lower bound on the LP relaxation of the MCDVRP, strengthened by two sets of valid inequalities, and (iii) solving the MCDVRP exactly using the results from steps (i) and (ii).

Step (iii) is done by solving the LP relaxation of the MCDVRP by column and cut generation. All subsets of the set of potential facilities that might be contained in any optimal solution are determined, using reduced cost criteria based on the solution of the LP relaxation, and the resulting problem is solved with a standard MIP solver.

Computational experiments are performed with the Perl, TB, B, ABR, PPW, and BMW instances. The algorithm clearly outperforms the previously discussed approaches, providing tighter lower bounds and solving most instances to optimality.

Contardo et al. (2013a) extend the work of Belenguer et al. (2011) and present three new arc-variable based formulations, namely, a three-index vehicle flow formulation as well as a two- and a three-index two-commodity flow formulation. They describe and prove seven new families of valid inequalities to strengthen the LP relaxations of the presented formulations and give heuristic and exact separation procedures. Computational experiments are performed with the ABR, B, Perl, PPW, and TB benchmark instances. The results show that the three-index formulations are generally stronger than the two-index counterparts. This, however, does not always translate into better optimization results: As the two-index formulations have fewer variables, larger branch-and-bound trees can be examined in a given time, and thus some instances are solved to optimality with the two-index formulations but not with the three-index formulation. The authors compare the results obtained with their formulations to those obtained by Baldacci et al. (2011). They observe that the latter is able to solve many more instances and attribute this to the tighter relaxations provided by path-variable formulations.

Contardo et al. (2013c) describe an approach similar to the one taken by Baldacci et al. (2011). It is also based on the path-variable formulation of Akca et al. (2009), consists of three stages, and has at its core the enumeration of relevant subsets of the set of potential facilities and the solution of the corresponding MCDVRPs: Assuming a given upper bound, in the first stage, the authors solve the two-index formulation of Belenguer et al. (2011), strengthened with the valid inequalities described in Contardo et al. (2013a), by branch-and-cut, forcing integrality only on the facility variables. The aim is to determine all subsets of potential facilities that might improve the upper bound. For each such subset, the second and third stage are performed. In the second stage, the linear relaxation of the corresponding MCDVRP is solved by cut-and-column generation, yielding a lower bound on the LRP solution restricted to that subset. In the third stage, column generation is used to enumerate all remaining columns (if any) whose reduced costs are not greater than the gap between the upper bound and the lower bound for the current subset. For the resulting set of columns, an integral solution is then determined with a standard solver.

Two features of the algorithm shall be pointed out. The first is that numerous additional inequalities are added to the arc-variable formulation of Belenguer et al. (2011) as well as the path-variable formulation of Akca et al. (2009). Many of these inequalities are new, and proofs of their validity as well as descriptions of the separation procedures are given. The second is that columns are generated solving a relaxed pricing problem where only cycles of length two are forbidden, in contrast to Baldacci et al. (2011), where elementary paths are generated.

Computational experiments are performed with the Perl, TB, B, ABR, PPW, and BMW benchmark instances. The algorithm clearly outperforms the results obtained with the branch-and-cut approaches by Belenguer et al. (2011) and by Contardo et al. (2013a). Compared to the approach of Baldacci et al. (2011), the algorithm yields tighter lower bounds and is able to solve two open instances.

Both the algorithm by Baldacci et al. (2011) and the one by Contardo et al. (2013c) are highly complex and sophisticated and use a large number of algorithmic and implementation refinements. Their success depends on the quality of the computed bounds and the ability to reduce the number of facility subsets that have to be considered.

4.2 Matheuristics

Matheuristics are hybrid approaches combining metaheuristics and mathematical programming (Maniezzo et al. 2010). A metaheuristic that does not solve at least a subproblem using a mathematical programming formulation is not considered a matheuristic in the following. Neither is an incomplete optimization approach that does not solve a mathematical program to optimality but terminates prematurely when a predefined gap is undershot or performs only partial enumeration of a branch-and-bound tree.

Prins et al. (2007) present an iterative two-stage heuristic. Lagrangian Relaxation (LR) and Granular Tabu Search (GTS) are combined. As initialization, a feasible solution is computed by a simple greedy heuristic. Then, in the first stage, each route of the current solution is aggregated into a supercustomer. This yields an SSCFLP, whose LR (the single-source assignment constraints are relaxed) is solved by subgradient optimization. An upper bound to the SSCFLP is determined by a repair procedure. Disaggregating the supercustomers leads to a new LRP solution. In the second stage, this solution is improved by solving an MCDVRP with a GTS heuristic where infeasible solutions that violate the facility capacities are allowed and penal-

ized. Then, information on the most frequently used edges is stored. If the current solution has not been improved in the last iteration, new routes are computed, where often-used edges are preferred.
Computational experiments are performed using the TB, B, and PPW instances. The heuristic is compared with the approaches of Tuzun and Burke (1999), Barreto et al. (2007), and Prins et al. (2006a, see Section 4.3). The experiments show that the procedure is capable of finding high-quality solutions in short time and, on average, yields better results than the other approaches.
Chen and Ting (2007) present a sequential two-stage heuristic for the standard LRP with heterogeneous vehicles. They decompose the problem into a location-allocation stage, where the facilities to be opened and the assignment of customers to facilities are determined, and a routing stage, where a VRP is solved for each opened facility. The first stage is tackled by solving an LR of an SSCFLP by subgradient optimization, similar to Prins et al. (2007). An upper bound to the SSCFLP is obtained by a greedy heuristic. In the second stage, the VRPs for each opened facility are solved by constructing a feasible solution with a nearest neighbor heuristic and improving this solution with Simulated Annealing (SA), using 2-opt, swap, and insertion moves and a reheating phase. After that, another SA heuristic is performed that allows swapping customers between routes of different facilities. Computational experiments are performed with the Perl and B instances. In numerical tests on the Perl instances, their method is able to improve on the results of Perl and Daskin (1984), Hansen et al. (1994), Wu et al. (2002), and Wang et al. (2005). On the B instances, they are able to clearly improve the results of Barreto et al. (2007) within fast run-times. Here, no comparison to Prins et al. (2006a) and Prins et al. (2006b) is given.
Özyurt and Aksen (2007) solve an LRP with already existing, uncapacitated facilities that incur costs for closing. The authors present an MIP formulation and develop a method based on nested LR: An outer LR embedded in subgradient optimization decomposes the parent problem into two subproblems. The first one is a kind of Facility Location Problem (FLP). It is solved to optimality with a standard solver. The second one resembles a capacitated and degree-constrained minimum spanning forest problem. It is NP-hard itself and cannot be solved (fast enough) with a standard solver. Thus, it is tackled again with LR by relaxing the degree constraints. It then becomes a minimum spanning forest problem with a minimum number of outgoing arcs at root nodes (facilities). Upper bounds are computed as follows. The solution of the first subproblem yields a set of opened facilities. As soon as a new distinct facility set is found in the course of the subgradient iterations, a Tabu Search (TS) algorithm is triggered to solve the MDVRP associated with that set, and a feasible solution to the parent problem is obtained. Its objective value is checked against the current upper bound on the optimal objective value of the parent problem. The TS algorithm uses a construction heuristic that is a mixture of an insertion and a nearest neighbor heuristic. It employs strategic oscillation by allowing intermediate infeasible solutions penalized in proportion to the violation of the vehicle capacity constraints. Four different Local Search (LS) neighborhoods are used.
Computational experiments are performed with the TB and self-generated random instances. For the TB instances, the results of Tuzun and Burke (1999) are improved by 3.8% on average. The gaps between the lower and upper bounds are 24% on average for the TB instances and below 10% for the random ones.
Lopes et al. (2008) present a decision-support tool and describe the embedded solution procedure, which is a sequential route-first location-second heuristic. A cluster-first route-second construction heuristic is used to determine routes. The authors apply four different clustering methods, using six proximity measures for points in the plane (customer locations), as described in detail in (Barreto et al. 2007). Routes for customer clusters are computed using a commercial solver on a two-index formulation with dynamic generation of subtour elimination constraints if the number of customers is 40 or less. Otherwise, a TSP tour for each cluster is computed with an insertion heuristic that determines the next customer to be inserted with a farthest-neighbor criterion and inserts the customer based on a savings criterion. The tour is then improved by 3-opt. For the location phase, the authors solve a capacitated location-allocation problem to

optimality with a commercial solver. Routes are collapsed into single customers. The cost of such a supercustomer is determined by a savings function.

Computational results are presented for the B instances. The average gap between the heuristic solution and a lower bound obtained with a two-index Integer Programming (IP) formulation is 4.8%.

Chen and Chen (2009) develop so-called approximation algorithms, which may be regarded as a special kind of matheuristic, for two slightly different types of standard LRPs. An α-approximate algorithm is a heuristic that guarantees to find a feasible solution, if one exists, and that furthermore guarantees that the objective function value of the solution found will be at most α times the optimal objective function value. The authors study two problems with uncapacitated facilities and heterogeneous, uncapacitated fleet (there is one vehicle per potential facility, and the costs of vehicles stationed at different facilities may be different). The underlying application background is not goods logistics but communications networks. Both problems consider fixed costs for opening a facility, variable costs for traversing an edge, and fixed costs for assigning a customer to a facility. In the first problem, these latter costs differ from facility to facility, but for each facility, the assignment costs of all customers are the same. In the second problem, these assignment costs may also differ between customers.

The authors develop two 12-approximate algorithms that exploit the relationship between (minimum) spanning or Steiner trees and closed tours (cycles) in graphs as described in the seminal works by Goemans and Williamson (1994) and Goemans and Williamson (1995). No computational experiments are performed.

Contardo et al. (2013b) present a matheuristic using a Greedy Randomized Adaptive Search Procedure (GRASP) and heuristic column generation. Their procedure consists of four parts. The first part is a GRASP that uses the Randomized Extended Clarke and Wright Algorithm (RECWA) by Prins et al. (2006a, see Section 4.3.1). The second part is an LS that uses seven local neighborhoods in a fixed order and in a cyclic fashion. The third part is an implementation of the MIP-based V3 neighborhood used by Pirkwieser and Raidl (2010, see Section 6). The fourth part is a large neighborhood search based on the same MIP. The third part solves the MIP using an initial set of customer sequences, the fourth part applies heuristic column and cut generation. Columns, i.e., customer sequences to be inserted into the existing route parts, are generated with a TS and a heuristic dynamic programming procedure that tries to add further customers to a column obtained with the TS. The valid inequalities from the three-index vehicle flow formulation presented in (Contardo et al. 2013a) are added. The GRASP and the LS are executed once, at the beginning of the overall algorithm. Then, the third and fourth part are repeated until a stopping criterion is met.

Computational experiments are performed with the Perl, TB, B, ABR, BMW, and selected PPW instances. The results compare favorably with those obtained by numerous previous authors (Prins et al. 2006a, 2007, Prodhon and Prins 2008, Duhamel et al. 2010, Pirkwieser and Raidl 2010, Yu et al. 2010, Hemmelmayr et al. 2012), yielding similar or tighter average gaps for all instance sets.

Lam and Mittenthal (2013) present a three-stage heuristic. In the first stage, a hierarchical clustering approach with single linkage as distance measure is used to find clusters that are feasible with respect to vehicle capacity. In the second stage, TSPs are solved for each facility and cluster using the Lin-Kernighan-Helsgaun heuristic (Helsgaun 2000). The resulting routing costs are taken as input for an FLP that is solved to determine the optimal facility configuration and cluster assignment. In the third stage, an LS with string-relocate and string-exchange moves using a first-improvement strategy improves the solution. Two different stopping criteria for the clustering are investigated, one based on vehicle capacity, the other on vehicle capacity and the within-cluster variation (Lam et al. 2009). In numerical studies on the TB and B instances, the solution quality of the proposed methods is clearly inferior to the comparison methods of Prins et al. (2006a,b, 2007). A direct comparison of run-times is not made, however, run-times of the proposed method are fast and stay below one minute even for instances with 150 customers and 20 facilities.

Escobar et al. (2013) propose a two-stage matheuristic. In the construction stage, an initial solution is generated as follows. First, the algorithm creates a giant tour that visits all customers by means of the Lin-Kernighan heuristic (Lin and Kernighan 1973), and the resulting tour is split according to the vehicle capacity in order to determine customer clusters. Next, an SSCFLP is solved exactly with a standard solver to obtain the optimal assignment of clusters to facilities. The cost of a cluster-facility combination is given by the TSP tour serving the cluster from the facility, again computed with the Lin-Kernighan heuristic. This process is repeated for each possible splitting of the initial tour. The best solution found undergoes a procedure that tries to save routing cost by dividing long routes and assigning one of the parts to a different facility. Subsequently, the routing solution of each facility is improved by a routine that uses a selection of the algorithms presented in the VRP library of Groër et al. (2010).
The improvement stage aims at simultaneously optimizing the routing solution for all facilities, i.e., an MCDVRP solution is considered. Infeasible solutions with respect to facility and vehicle capacities are allowed and handled by a penalty mechanism. First, the algorithm attempts to reduce the number of routes by removing the least loaded routes and inserting the customers into the remaining routes at the best position. The routing solution over all facilities is then improved by a GTS (Toth and Vigo 2003) using five neighborhoods: insert, swap, 2-opt, sequence relocate with two customers and sequence exchange with two customers. The sparsification graph of the GTS is defined based on the average cost of an edge in the currently best solution. Only moves whose involved edges are all in the sparsification graph are considered. To diversify the search, the authors (i) alter the sparsification graph, (ii) use a dynamic penalty approach, and (iii) inspect the best facility-capacity feasible solution found after a predefined number of iterations, apply the Groër et al. (2010) improvement procedure to this solution, and continue the search from there. Finally, when the algorithm has found no improving move for a given number of iterations, the solution is perturbed by a CROSS-Exchange similar to the one described in Renaud et al. (2004).
In numerical studies, the authors show that each of the algorithmic components contributes to the success of the method. The algorithm is compared to the methods of Prins et al. (2006a,b, 2007), Duhamel et al. (2010) and Yu et al. (2010) on the TB, B, and PPW instances. Over all test instances, the algorithm finds the highest number of Best Known Solutions (BKS) of all methods and the average deviation from the BKS is smallest. Considering the test sets separately, the heuristic ranks first concerning solution quality on the TB instances, second on the PPW instances and third on the B instances. The run-time of the algorithm is clearly faster than that of the two main competitors regarding solution quality, namely Duhamel et al. (2010) and Yu et al. (2010).
Alvim and Taillard (2013) propose a solution method for large-scale LRPs with uncapacitated facilities and capacitated vehicles. Each customer location serves as a possible facility location, so that the size of a problem is defined by the number of customer locations n. The algorithm is based on the Partial Optimization Metaheuristic Under Special Intensification Conditions (POPMUSIC) framework, whose general idea is to locally improve parts of a given solution (subproblems) in order to improve the overall solution (see Taillard 1993, Taillard and Voß 2001). The strength of this framework lies in the algorithmic complexity of improving an initial solution, which is quasilinear in problem size and the maximal number of parts considered to form subproblems (in this work, routes are defined to be problem parts). Therefore, the main goal of the method is to quickly find an initial solution of good quality. To this end, the authors propose a matheuristic with run-time complexity of $O(n^{3/2})$ that is based on a gradient method for solving the LR (on the vehicle capacity constraints) of a Capacitated p-Median Problem (CPMP).
More precisely, the algorithm first determines a number of centers by solving the relaxed CPMP for a sample of the customer locations. Next, all customers are assigned to the selected centers and the resulting superclusters are decomposed into clusters based on the vehicle capacity by again solving the relaxation of the CPMP with the gradient method. The gradient method is enhanced by an LS that aims at reducing capacity violations of clusters by moving customers to

their second nearest cluster if the latter still has available capacity. For the resulting clusters, TSP tours are determined and all tours with a length greater than the opening cost of a facility are split. Then, the algorithm opens a facility for each cluster and greedily tries to merge facilities. Finally, a feasible MDVRP solution is obtained by dividing tours that do not meet vehicle capacity constraints into two tours. As improvement method in the POPMUSIC framework, a TS similar to the one described in (Taillard 1993) is used.
Numerical experiments are conducted on problem instances with 17,237–1,904,711 customer locations derived from the world TSP (`http://www.tsp.gatech.edu/world/index.html`). The proposed method is extremely fast: It is able to find a solution to the biggest instance in less than 14 hours. To assess the solution quality of the method, tests on large VRP instances proposed by Li et al. (2005) and Zachariadis and Kiranoudis (2010) are conducted, and results are within 10% of the BKS with a computation time below 10 seconds.

4.3 Metaheuristics

To better structure this section, we introduce subsections that distinguish between problems with capacitated and uncapacitated vehicles.

4.3.1 Standard LRPs with capacitated vehicles

Chan and Baker (2005) study the standard LRP with uncapacitated facilities. With respect to the vehicles, two cases are considered: Vehicles either have only route length restrictions or, additionally, restricted loading capacities. The authors present an arc-variable based MIP and a two-stage heuristic. In the first stage, all facility subsets with a cardinality between a lower and an upper bound are enumerated. For each subset, a minimum spanning forest containing the facilities in the subset and all customers is determined by assigning a customer to its closest facility. In the second stage, vehicle routes for each opened facility are computed with a savings heuristic.
Computational experiments are performed with real-world data from document delivery. For small instances with 1–2 facilities and 5–6 customers, the heuristic finds solutions with a gap of less than 4% to the optimal solutions determined by solving the MIP model exactly with a standard solver. A case study for a larger real-world instance with 12 potential facilities and 181 customers is presented, and practical considerations of opening and closing facilities are discussed.
Bouhafs et al. (2006) propose a hybrid of SA and Ant Colony Optimization (ACO). The algorithm iteratively applies SA for the selection of facilities to open and ACO for determining the routes. The approach is compared to that of Barreto et al. (2007) on a random selection of the Perl and B instances. For the selected instances, the quality of the proposed method is on a par with or better than the solutions obtained by Barreto et al. (2007). No run-times are reported.
Prins et al. (2006a) present a GRASP that is enhanced by a learning component and uses Path Relinking (PR) as post-optimization step. In the GRASP, a Randomized Extended Clarke and Wright Algorithm (RECWA) is used to construct solutions. RECWA first assigns customers to their closest facility with sufficient capacity and closes the facilities with no assigned customers. To merge the dedicated routes from facilities to customers, the traditional Clarke and Wright Algorithm is extended to evaluate the possibilities of assigning the merged route to different facilities: the facility of the first route, the facility of the second route, or any other facility. Randomization is achieved by selecting the route merge to be executed randomly among the list of mergers with the best savings. During the search, the size of this list is randomly varied within a given interval. The construction of a solution is finished if no merger can be found that leads to a cost reduction and is feasible with respect to facility and vehicle capacity. The routes of the resulting solution are subsequently improved by an LS comprised of customer relocate, swap, and a 2-opt move adapted to the case of multiple facilities. The LS executes the first improving move and stops if no improving move can be found.

A diversification and an intensification phase are used within the RECWA. Both phases are based on restricting of the facilities that can be used for the initial customer assignment. In the first iteration of the diversification phase, all facilities may potentially be opened. In the next iterations, initially only two facilities are available: The first one is iteratively selected so that each facility was opened at least once, the second one is randomly drawn from the remaining ones. If a customer cannot be assigned because of a capacity violation, his closest facility is added. In the intensification phase, only the facilities that are open in the best solution found in the diversification phase may be used. Here not only the assignment of customers is restricted by this subset but also the merge operations during the RECWA.
In a post-optimization step, PR is used to generate new solutions from an elite set. The elite set is constructed from a set of the best solutions found in the diversification phase. First, the best solution is added to the elite set, then iteratively the most distant solution to those already added until a predefined size is reached. The distance calculation is based on the similarity of the facility configurations and the routing parts of two solutions. New solutions are generated by applying PR to all pairs of solutions in the elite set, using both elements of a pair as the guiding solution.
Computational tests are conducted on the TB, B, and PPW instances and the proposed method improves the solution quality compared to the methods of Tuzun and Burke (1999) and Barreto et al. (2007) on all instances. Run-times are competitive on instance sets B and PPW, while on TB, the method of Tuzun and Burke (1999) is clearly faster.
Prins et al. (2006b) propose a Memetic Algorithm with Population Management (MA|PM) for the same problem. MA|PM is characterized by a small population that is initially generated by means of the RECWA of Prins et al. (2006a) and a randomized nearest-neighbor-based method, both improved by LS. Periodically, a new population is generated keeping only the best solution of the previous population. The algorithm encodes the vehicle routes as a concatenation of giant tours without route delimiters for each facility, denoted as customer sequence part. In the other part of a chromosome, the facility status part, for each facility an index is stored indicating the position in the customer sequence part at which the giant tour assigned to the facility starts. If a facility is closed, a zero is stored instead of the index. The fitness of an individual is determined based on an optimal partitioning of the giant tours of each facility into vehicle routes by means of the splitting procedure of Prins (2004).
Binary tournament selects the first parent from a subset of elite solutions and the second from the entire population. Standard one-point crossover is applied on the facility status part and a variant of one-point crossover for path representations on the customer sequence part. The crossover is followed by a repair mechanism to render solutions feasible with regard to facility capacities. An LS similar to the one in (Prins et al. 2006a) and a reduced, fast version of the same LS are applied to the generated offspring with given probabilities. A population management step, whose goal is to produce a large diversity of the population, substitutes the common mutation step. The diversity is guaranteed by only accepting a new solution if its distance to the solutions in the population is above a given threshold. The same distance measure proposed in (Prins et al. 2006a) is used. Offspring that do not exceed the threshold are discarded. Otherwise, the generated offspring replaces the solution with the lowest quality in the current population.
The method is evaluated on instance sets TB, B, and PPW and compared to the methods of Tuzun and Burke (1999), Prins et al. (2004, 2006a, 2007) and Barreto et al. (2007). Concerning the solution quality, MA|PM is able to roughly match the solution quality of the best comparison method (Prins et al. 2007) for all instance sets, but with clearly higher run-times. Detailed results for the MA|PM can be found on the website `http://prodhonc.free.fr`.
Duhamel et al. (2008) propose another MA. To generate the initial solutions, three classical heuristics and the RECWA of Prins et al. (2006a) are used. Contrary to the two-part encoding used in Prins et al. (2006b), a giant-tour without route or facility delimiters is used to represent a complete solution. The fitness evaluation is based on a splitting procedure that uses a dynamic-programming based labeling algorithm to calculate the optimal solution subject to vehicle capacity, fleet size, and facility capacity constraints, similar to the one described in Prins

(2004). To speed up the computation of the fitness, dominance rules avoiding the generation of unnecessary labels are introduced. The authors observe that the giant tour representation avoids repair procedures and enables the use of classical crossover and LS procedures. No details are given on these components.
In numerical tests on the TB and PPW instances, the quality of the proposed method is found to be slightly better than that of Prins et al. (2006a) but not able to match the quality of Prins et al. (2007, 2006b). On the B instances, the proposed MA provides the best solution quality of all methods. However, for all test instances, the MA requires significantly higher run-times than the comparison methods.
Duhamel et al. (2010) propose a GRASP approach that calls an Evolutionary Local Search (ELS). The key ingredients of the algorithm are: (i) It searches within two solution spaces, namely giant tours without trip or facility delimiters and LRP solutions, and (ii) it uses a tabu list with forbidden facilities that is used to diversify and intensify the search. Initial solutions are constructed by the RECWA of Prins et al. (2006a) and are improved by the LS method described in the same work. The resulting solution is then transformed into a giant tour representation. In the next step, a number of ELS iterations are carried out, where in each iteration a prespecified number of child solutions are generated by a so-called hub mutation operator. More precisely, the operator changes the tabu list, and the different children are defined by different tabu lists. To create the tabu lists, the operator removes a randomly selected facility from the current tabu list and selects one facility that is not on the tabu list. If without the selected facility still enough facilities are open to serve the total customer demand, the selected facility is added to the tabu list with a given probability. Next, for each child, the mutation on tour operator swaps two randomly selected nodes in the giant tour representation.
The resulting giant tour is transformed back to an LRP solution by means of a splitting procedure (inspired by the one presented in Prins 2004) that minimizes the total cost and respects vehicle capacity, fleet size, facility capacity constraints, and the tabu list for the facilities. For tackling large problems with more than 100 customers, a heuristic version of the splitting procedure is applied. Finally, the LRP solution is again improved by the LS. If one of the generated child solutions improves the starting solution, it becomes the starting solution for the next ELS step. The algorithm ends after a given number of GRASP iterations.
The hybrid is tested on the TB, B, and PPW benchmark instances and compared with the methods of Prins et al. (2006a,b, 2007). The algorithm is able to improve a large number of best solutions and achieves improved average solution quality on most benchmark sets; however, it requires clearly higher run-times than all comparison methods.
Marinakis and Marinaki (2008a) develop a hybrid of Particle Swarm Optimization, GRASP, Expanding Neighborhood Search (ENS, see Marinakis et al. 2005) and PR. The authors conduct tests on the Perl and B instances and are able to compete with the solution quality of Barreto et al. (2007), while requiring moderate run-times. **Marinakis and Marinaki (2008b)** provide a bilevel programming formulation for the same problem and propose a Genetic Algorithm (GA) as solution method. The GA uses the GRASP and ENS procedure described in (Marinakis and Marinaki 2008a) for generating the initial population, a crossover inspired by Adaptive Memory Programming (Taillard et al. 2001) and a mutation operator that opens or closes facility locations. A case study of a company in Greece selling wood products is discussed. In tests on the Perl and B instances, the method is able to slightly improve on the results of Barreto et al. (2007), but no run-times are reported. Both works (Marinakis and Marinaki 2008a,b) make no comparison to the previously published methods of Prins et al. (2006a,b) or to each other.
Sahraeian and Nadizadeh (2009) propose a clustering heuristic combined with an ACO algorithm for the routing phase. The method shows acceptable solution quality on a selection of the B instances but requires rather long run-times. **Nadizadeh et al. (2011)** present an extension of this method, and results on all B instances are provided, but no run-time information is given. The solution quality of the algorithm is competitive with the methods of Bouhafs et al. (2006), and Prins et al. (2006a, 2007) and is able to outperform the methods of Barreto et al. (2007) and Marinakis and Marinaki (2008b). **Jokar and Sahraeian (2011)** introduce a multi-

start LS and conduct tests on the B and PPW instances. The algorithm achieves acceptable quality but is not able to compete with the methods of Prins et al. (2006b, 2007). No run-times are provided. **Jokar and Sahraeian (2012)** present an SA with better quality on the same selection of B instances as Sahraeian and Nadizadeh (2009). Again, no run-times are reported.
Yu et al. (2010) present an SA algorithm that uses a fixed-length string of facility and customer identifiers and dummy zeros to represent an LRP solution. The string starts with a facility. An open facility is followed by at least one route (represented as customer sequence), and a route ends if either a dummy zero or the next facility appears or if the vehicle capacity is exceeded. Facility capacity violations are possible and are penalized. The authors generate an initial solution in a greedy fashion by choosing facilities and assigning customers based on their closeness. Subsequently, TSP tours are created for each facility with the Lin-Kernighan heuristic and the resulting tours are split according to vehicle capacities. As neighborhood operators, insertion, swap, and 2-opt are defined on the solution representation, i.e., different cases depending on the type of the elements (facility or customer) affected by the move are considered. All moves are applied with the same probability. Solutions violating vehicle capacities are repaired by dispatching an additional vehicle, facility capacity violations are handled as described above, and solutions whose representation does no longer start with a facility are simply discarded. After each temperature reduction, the SA method applies a greedy LS to the currently best solution.
The performance of the proposed method is investigated on the TB, B, and PPW instances. On all test sets, the SA matches a large portion of the BKS at the time and improves several BKS. Moreover, the SA dominates all comparison methods (Tuzun and Burke 1999, Bouhafs et al. 2006, Prins et al. 2006a,b, 2007, Barreto et al. 2007, Duhamel et al. 2010) in terms of average solution quality. Run-times of the algorithm are acceptable for a strategic problem but are clearly higher than those of some of the comparison methods.
Derbel et al. (2011) propose a Variable Neighborhood Search (VNS) algorithm that embeds a Variable Neighborhood Descent (VND) as LS component. The shaking step repeatedly applies customer insertion moves in an intra-route and an inter-route fashion. The customer to be inserted is selected randomly and inserted at the best possible position. For the VND, the four additional neighborhood structures customer swap, sequence insert with and without inversion, sequence exchange with and without inversion, and route exchange between facilities are used. The VND uses a first-improvement strategy in each iteration. Infeasible solutions with respect to facility and vehicle capacity are handled by means of penalty costs. In the numerical studies, the proposed VNS is compared to the methods of Prins et al. (2006b), Duhamel et al. (2010) and Yu et al. (2010) on the B instances and is able to obtain competitive results with respect to solution quality. No run-times are reported.
Jabal-Amelia et al. (2011) propose a VND algorithm. The initial solution is generated with a savings heuristic and seven neighborhood structures (four affecting the facility configuration and three affecting the routing) are applied in the VND to improve the solution. Computational tests are performed with the TB and B instances and show that the algorithm is not able to match the quality of the methods of Prins et al. (2006a, 2007) and Yu et al. (2010). No run-times are reported.
Stenger et al. (2011) present a metaheuristic hybrid of GRASP and VNS. First, a GRASP using the RECWA of Prins et al. (2006a) and a greedy LS with 2-opt intra-route and relocate/exchange inter-route moves is executed. Infeasible solutions are handled by a dynamic penalty mechanism. To diversify the search, the best three solutions of the GRASP are kept as input for the further steps. Next, the routing of the solutions is improved by a short VNS run. The VNS is based on the work of Stenger et al. (2013) and addresses an MDVRP. The shaking step uses a set of 36 neighborhoods defined by the cyclic exchange operator. The LS uses the moves described above and solution acceptance in the VNS is based on an SA criterion. The goal of the short VNS run is to further enhance the accuracy of the evaluation of the facility configuration. The authors find that the (necessarily) imprecise routing solutions in tendency lead to facility configurations with too many open facilities. To counteract this point, they apply

a facility reduction phase that investigates a given number of feasible facility configurations with one less facility open than in the currently best solution. Finally, the routing of the best three solutions found in the course of the algorithm is further improved by an extensive VNS run.
In numerical studies, the authors show that both the VNS component and the facility reduction mechanism significantly contribute to the quality of their method. The hybrid is compared to the algorithms of Prins et al. (2006a), Duhamel et al. (2010) and Yu et al. (2010) on the TB, B, and PPW benchmark instances based on its best solution in 10 runs and the average run-time. In this way, the proposed algorithm is able to outperform the other methods concerning solution quality and run-time. Moreover, a high number of new BKS is found.
Ting and Chen (2013) develop an ACO algorithm that iteratively applies three ant colonies addressing facility location, customer assignment and routing. The first colony opens a random number of facilities and selects them based on pheromone information and the ratio of facility capacity and opening cost. The second colony assigns customers to the opened facilities based on pheromone information and the distance of customers to facilities and subsequently employs a repair mechanism to ensure that facility capacities are respected. Thus, the first two colonies together generate an FLP solution, and for each solution generated, the third ant colony runs separately to generate a VRP solution for each open facility and its assigned customers. Tours are generated based on savings and pheromone information and the best solution of each iteration is improved by an LS consisting of customer insertion, swap, and 2-opt moves. The resulting LRP solution of each global iteration is improved by an LS method with best-improvement strategy that applies customer insertion and swap in an MDVRP context, i.e., over all routes of all facilities.
All colonies keep their own pheromone matrices and employ different updating rules. The first two colonies cooperate via information exchange using the global pheromone update. The global update is based on the overall best solution and best solution found in the current iteration. This mechanism aims at finding a good tradeoff between exploration and exploitation and is also employed in the global pheromone update of the third colony, where the iteration-best and overall-best tour are used.
The algorithm is able to match or improve the former BKS for all Perl instances, however, one has to note that the Perl instances have not been used as test instances by the high-quality algorithms of recent years. On the B instances, the ACO is able to match the solution quality of Duhamel et al. (2010) and improve on the quality of Yu et al. (2010). On the PPW and TB instances, the method is able to improve on both competitors. The run-times of the algorithm are reasonable.
To summarize, we point out that the best solutions on the standard test instances of TB, B, and PPW are currently achieved by the following math- and metaheuristics specifically developed for the standard LRP: Duhamel et al. (2010), Yu et al. (2010), Stenger et al. (2011), Escobar et al. (2013), Contardo et al. (2013b), and Ting and Chen (2013). The differences in solution quality of these methods are close to negligible and all methods have run-times that are clearly adequate to provide decision support for a strategic problem. Note that several methods developed for more general LRP variants also achieve very convincing performance on benchmark instances of the standard LRP (see, e.g. Nguyen et al. 2010, Pirkwieser and Raidl 2010, Hemmelmayr et al. 2012).

4.3.2 Standard LRPs with a single uncapacitated vehicle

The following works study an LRP where a single, uncapacitated vehicle is considered for each facility. This LRP was originally introduced by Albareda-Sambola et al. (2005).
Derbel et al. (2010) present an Iterated Local Search (ILS). The LS component applies the first improving move of four neighborhoods (exchange and insert as intra- and inter-route moves) in a sequential fashion until no improving solution can be found. Only open facilities are considered in the LS. The facility configuration is modified in the perturbation step by randomly closing and opening a facility. Capacity violations can be handled by a penalty mechanism, but a very

high penalty factor is used in the numerical studies so that practically only feasible solutions are allowed. The algorithm is tested on the ADF instances and is able to improve on the solution quality of the TS presented in Albareda-Sambola et al. (2005) for almost all test instances. The run-time of the ILS is clearly below that of the comparison method.
Derbel et al. (2012) develop a GA hybrid that uses the above described ILS within a memetic approach. Solutions are encoded with a two-part chromosome: The first part encodes the assigned facility for each customer, while the second part describes the overall order in which customers are visited. The initial population is generated in a random fashion and individuals are selected for crossover based on a probability that depends on their solution rank. The authors apply standard one-point crossover to the first part of the chromosome and a variant of one-point crossover adapted to the path representation to the second part. Mutation randomly changes the facility assignment of one customer in the first part and randomly moves one customer to a different position in the second part. If the generated individual is only a certain threshold worse than the currently best solution, it is improved by an ILS step. If the resulting solution is better than the worst solution in the population, it replaces this solution. In numerical tests using the ADF instances, the hybrid is able to improve on the pure ILS considering average solution quality. However, the GA has higher run-time and is not able to match the quality of the pure ILS on the larger instances.
Jarboui et al. (2013) propose several VNS algorithms that all use a VND as LS component. They propose five neighborhoods for the VND. The first four are customer-related (insert, swap, sequence insert and insertion of an inverted sequence) and are used in both intra- and inter-route fashion. The fifth neighborhood closes one facility and opens another in a random manner. As shaking steps, the authors investigate repeated moves in the insert neighborhood, the facility swap neighborhood, and in a combined neighborhood of customer insert and facility swap. Moreover, shaking can be carried out in completely random fashion or in so-called semi-random fashion, i.e., the customer (or route in the case of the facility swap move) is selected at random and inserted in the best position (cp. Derbel et al. 2011, Section 4.3.1). In numerical studies, the authors find that the combined shaking step using the semi-random strategy shows the best results considering the trade-off between solution quality and run-time. On the ADF instances and the B instances without vehicle capacities, the VNS is able to outperform the ILS of Derbel et al. (2010) with respect to solution quality and run-time. On the benchmark sets TB and PPW without vehicle capacities, the VNS achieves better solution quality but has a slightly higher run-time than the ILS.

5 Multi-echelon LRPs

Multi-echelon LRPs have only very recently attracted the interest of researchers: In the last survey on LRPs by Nagy and Salhi (2007), just three references are mentioned. However, judging from the number of publications that have appeared since then, multi-echelon LRPs are the most important LRP modeling extension of the last decade. Although the literature on the topic dates back to Jacobsen and Madsen (1980), the terms multi-echelon or N-echelon VRP/LRP are, in fact, first used in (Gonzalez Feliu et al. 2008) and (Perboli et al. 2008b) (of which an extended version with new results was later published as (Perboli et al. 2011)).
This development starts to reflect the practical importance of multi-echelon LRPs. Applications include strategic or tactical design of national or international consumer goods distribution networks, postal and parcel delivery distribution systems, press distribution, grocery distribution, home delivery services, e-commerce, and multimodal transportation (see Gonzalez Feliu 2009). In particular, city logistics concepts for operational planning of goods distribution on the local level, which have been discussed in practice for decades, have finally been investigated by several researchers (see below).
Most papers studying multi-echelon LRPs are concerned with the two-echelon case and ignore temporal aspects such as time windows or the synchronization of transshipments at intermediate levels. These papers are reviewed in the following subsection. After that, a subsection is devoted

to works on two echelon LRPs (2E-LRPs) with temporal aspects. Finally, Subsection 5.3 deals with N-echelon problems for $N > 2$.

5.1 Two-echelon LRPs

Perboli et al. (2011) present an arc-variable based MIP formulation for the 2E-LRP with one level-0 facility and capacitated level-1 facilities without fixed opening costs, but with variable, facility-specific costs that depend on the amount of load transshipped at a facility. The formulation was introduced in (Gonzalez Feliu et al. 2008). The authors derive additional valid inequalities and develop two matheuristics based on the formulation. Both heuristics exploit the fact that, once the assignment of customers to level-1 facilities is fixed, the problem decomposes into one VRP for the level-0 facility and one VRP for each level-1 facility. Given an optimal solution to the LP relaxation, the first heuristic performs a diving procedure (branch-and-bound without backtracking). It successively fixes to zero the binary customer-to-facility assignment variables with smallest positive value and highest pseudocost (see Linderoth and Savelsbergh 1999) and then solves the resulting LP relaxation again. Once all assignment variables are integral, the resulting CVRP instances are solved by means of the heuristic introduced in (Perboli et al. 2008a). A repair routine is included to ensure that the capacity constraints of the level-1 facilities are satisfied. The second heuristic relaxes the integrality requirements on the arc variables (but not on the assignment variables). A small set of constraints is added to make sure that level-1 capacities are maintained. A standard solver is run on the resulting simplified MIP for a given time limit, and the k best feasible solutions found are stored. For each of these solutions, the resulting CVRPs are solved with the heuristic of Perboli et al. (2008a).
Computational experiments are performed with the GPTV instances and the 50-customer CPMT instances. The results show that, even when the valid inequalities are added, the MIP formulations of the GPTV instances can be consistently solved to optimality with a commercial solver only for instances with up to 21 customers. Only one instance with 32 customers is solved to optimality. For larger instances, gaps of between 0.7% and 42% remain after three hours of computation time. For the CPMT instances, no optimal solutions are found, and the gap averages 11%. The results of the heuristics show that the second one outperforms the first one on the CPMT instances, whereas on the GPTV instances, no heuristic dominates the other. Although computation times are generally below 30 seconds, both heuristics find better solutions than the best upper bounds provided by the exact method for many of the larger instances.
Perboli et al. (2009) and **Perboli et al. (2010)** develop valid inequalities for the formulation introduced in (Gonzalez Feliu et al. 2008) and show the effectiveness of these cuts by performing computational experiments with the GPTV and the 50-customer CPMT instances. Using the new cuts, all GPTV instances with 32 customers or less are solved to optimality, and for the remaining instances of both sets, the gaps are significantly reduced.
Crainic et al. (2008) and **Crainic et al. (2010b)** study heuristics for a 2E-VRP with one level-0 facility and limited fleet. However, the potential facilities neither incur fixed costs for opening, nor is the number of facilities to be opened smaller than the number of available facilities, nor are the facilities capacitated. Thus, according to our LRP criteria given in the Introduction, their problem is not an LRP. Nevertheless, the papers are described here because they provide approaches and insights as well as benchmark instances that the authors use in subsequent publications on 2E-LRPs.
Crainic et al. (2008) separate the problem into two subproblems, one for each echelon. The second-echelon problem is solved first, and from its solution, a problem instance for the first echelon is constructed, where the level-1 facilities act as customers with a demand equal to the sum of the demands of the customers assigned to each facility in the second-echelon problem. Two constructive and three LS improvement heuristics are presented. The first constructive heuristic performs an initial clustering of customers by assigning each customer to its nearest level-1 facility, taking into account the limited number of available vehicles. Then, the resulting VRPs on the second and first echelon are solved by means of a commercial heuristic VRP solver.

After that, the authors try to improve the solution by moving one customer from its assigned facility to the next closest one. If such a move is feasible (with respect to the limited fleet), the resulting VRPs for the two affected level-2 facilities and the first echelon are solved again. This is repeated until no improved solution is found or a stopping criterion is met. The second constructive heuristic uses the commercial heuristic solver to solve an MDVRP with limited fleet for the second echelon and then for the first echelon. The three LS improvement heuristics are to (i) split a route into two routes if it contains a pair of successive customers whose distance lies above a threshold, (ii) move one customer from one route to another, and (iii) swap two customers between two routes. If necessary, the level-1 subproblem is re-solved.
Despite the chronological inversion, computational experiments are performed with the GPTV and the CPMT instances with 50, 100, and 150 customers. For the GPTV instances, the clustering heuristic clearly outperforms the MDVRP-based one. On the larger GPTV instances, the clustering heuristic, in combination with the improvement procedures, yields better solutions than the exact procedure used in Perboli et al. (2011). Computation times are a few seconds for instances with 32 customers. The clustering heuristic with improvement is then employed to perform analyses on the impact of the geographical distribution of level-1 facilities and customers, using the CPMT instances.
Crainic et al. (2010b) continue this research, again using the clustering heuristic with improvement. Extensive studies are performed with different configurations regarding the locations of facilities and customers. Both Crainic et al. (2008) and Crainic et al. (2010b) conclude that, if the facilities are adequately located, a two-echelon system can significantly reduce the total distribution costs compared to a one-echelon system with one central facility. They state that their results emphasize the potential benefit of two-echelon systems for city logistics in large urban areas but point out the need for further research, especially on extensions of the 2E-VRP model used.
Crainic et al. (2010a) present a multi-start heuristic to solve the 2E-LRP with one level-0 facility and capacitated level-1 facilities. Perturbed solutions for the multi-start component are constructed by two stochastic rules specifying probabilities for the assignment of a customer to a facility and performing roulette-wheel selection. To compute an initial solution and to perform an LS on the perturbed solutions, the authors use the clustering heuristic and the improvement procedures from Crainic et al. (2008). To account for facility capacities, the authors introduce a repair procedure that tries to move customers one by one from overloaded facilities to others with free capacity.
Computational experiments are performed with the GPTV and the CPMT instances with 50 customers. The results show that the heuristic outperforms the two matheuristics described in (Perboli et al. 2011), yielding a better solution quality in less computation time.
Sterle (2010), **Boccia et al. (2011)**, and **Crainic et al. (2011b)** present MIP formulations for a problem version with several level-0 facilities, where both level-0 and level-1 facilities are capacitated and incur fixed opening costs. The latter two papers are technical reports that summarize the results described in the first one, which is the author's Ph.D. thesis. Four different formulations are presented. The first one is a three-index arc-variable formulation based on the one introduced in (Ambrosino and Scutellà 2005) for the three-echelon case. The second one is a two-index arc-variable formulation based on the MDVRP formulation described in (Dondo and Cerdá 2007), the third one extends the two-index arc-variable standard LRP formulation by Prins et al. (2006a), and the fourth one is a path variable formulation.
Computational experiments are performed with the first and second formulation using a commercial solver on the smaller S instances with up to 4 level-0 facilities, 10 level-2 facilities, and 25 customers. The largest instances solved to optimality have 3 level-0 facilities, 8 level-2 facilities, and 10 customers. The objective function value of the root node LP relaxation is always much higher in the three-index formulation, and in all but two cases, the three-index formulation finds solutions as good as or better than those found with the two-index formulation. For the larger S instances, which have up to 5 level-0 facilities, 20 level-2 facilities, and 200 customers, a heuristic sequential decomposition is performed: First, a two-echelon Capacitated FLP (CFLP) is solved,

and then two MDVRPs, one for each echelon. The three subproblems are solved (to optimality or until a prespecified maximum gap) with the commercial solver. Using this decomposition approach, feasible solutions are computed for all instances.

Sterle (2010), **Boccia et al. (2010)**, and **Crainic et al. (2011a)** present a TS heuristic for the 2E-LRP version just described. Again, the latter two papers are technical reports that summarize the results described in the first one. Also in these works, the problem is decomposed by echelon, and both subproblems are further decomposed into a CFLP and an MDVRP. Essentially, the TS algorithm used by Tuzun and Burke (1999) for the single-echelon LRP is extended to the two-echelon case. In a first step, a solution to the two-echelon CFLP is determined by means of a greedy heuristic. Given the initial CFLP solution, a route plan is computed for each facility with the savings heuristic and 2- and 3-opt improvements. Then, shift and swap moves are performed (shift a customer from a route to another one and swap two customers between two routes). These moves are executed first only between routes belonging to the same facility, and then between routes from different facilities. In the TS, on both echelons, besides the shift and swap routing moves, two move types affecting location decisions are performed (swap: close an opened and open a closed facility; add: open a closed facility).

Two move evaluation criteria are used, similar to the nested approach first proposed in (Nagy and Salhi 1996) and applied in (Tuzun and Burke 1999). To evaluate the first feasible solution and the location moves, only the fixed facility opening costs and the direct distance between a customer and the assigned facility (or the direct distance between a level-1 facility and the assigned level-0 facility) are considered. To evaluate routing moves, the fixed opening costs and the exact costs of the routes are used. Moreover, a feedback loop between the two levels is included: When a new solution for the second echelon is determined that improves the best one found so far or violates the capacity of a level-1 facility, the subproblem for the first echelon is re-solved.

Computational experiments are performed with the S instances. For the smaller instances, the performance of the TS algorithm is compared to the results obtained with the branch-and-cut approach described in Sterle (2010) (see above). For those smaller instances for which an optimal solution could be found with the branch-and-cut algorithm, the TS always finds an optimal solution, too. For those instances where branch-and-cut failed to find an optimal solution, the TS always finds a better solution than the best feasible one found with branch-and-cut. For the larger instances, the TS with feedback loop clearly outperforms the one without in the large majority of cases.

Jin et al. (2010) study a 2E-LRP with several uncapacitated level-0 facilities, capacitated level-1 facilities, fixed opening costs for facilities on both levels, direct transports on the first echelon, and routing decisions with a heterogeneous fleet on the second. The authors present a three-index arc-variable formulation for the problem and develop a GA to solve it.

A solution is encoded in a fixed-length chromosome, using a scheme containing binary and general integer genes. A proportional selection scheme is used to choose individuals for reproduction, which is performed with a two-point crossover, and a random mutation operator is applied. A fixed-length first-in-first-out tabu list is maintained that stores parts of the chromosomes/solutions selected for reproduction.

Computational experiments are performed with two self-generated random instances with 5 level-0 facilities, 10 level-1 facilities and 20 and 40 customers respectively.

Nguyen et al. (2010) present a multi-start heuristic for the 2E-LRP with one level-0 facility and capacitated level-1 facilities with fixed opening costs. Using a vehicle incurs fixed costs, and it is allowed to visit customers on tours starting at the level-0 facility. The heuristic consists of two components: A GRASP and an embedded ELS/ILS. In both the GRASP and the ELS/ILS, a tabu list is maintained. The GRASP uses three construction heuristics followed by a VND. In the ELS/ILS, a prespecified total number of child solutions is created. The ELS/ILS first creates a giant tour consisting of all customers. The giant tour is initialized with the customers of a second-echelon tour. Then, the other second-echelon tours are successively inserted into the giant tour. After that, a mutation operator changes the sequence of customers on the giant tour.

Then, the currently opened level-1 facilities plus one currently closed facility are used to form a set of potential facilities. With this set, a split procedure generalizing the one presented in Prins (2004) is called that builds second-echelon tours from the giant tour. The second-echelon tours originate at a facility from the current set of potential facilities. If facility capacities are violated, a repair procedure is called. Finally, a VND scheme that only works on the routing part of a solution is executed. The whole process is iterated until a stop criterion applies.
The procedure is quite complex and contains a number of parameters that have to be set. It is noteworthy that, to find good parameter values, the performance of different setups is compared by sophisticated statistical tests. This goes beyond what is usually done in the literature for this purpose. Computational experiments are performed with the NPP-N, NPP-P and PPW instances. On the first two instance classes, the heuristic is compared with two unpublished procedures by the same authors and achieves very convincing results. On the third class, which consists of standard LRP instances, the heuristic is compared with existing, specialized procedures for the standard LRP (Prins et al. 2006a,b, 2007, Duhamel et al. 2010). The results show that the heuristic is competitive with the other approaches both with respect to solution quality and computation time.
Nguyen et al. (2012b) study the same variant as Nguyen et al. (2010) and present a multi-start ILS with a tabu list and PR. The authors give a three-index arc-variable formulation for the problem. The ILS cyclically uses the three heuristics described in Nguyen et al. (2010) to provide a good initial solution. The search then operates on two search spaces, namely, valid 2E-LRP solutions and TSP-like giant tours over the level-0 facility and all customers: After an LS on the 2E-LRP solutions obtained by the constructive heuristics, a giant tour is formed. A kick operator selected randomly out of three possible ones is then applied to perturb the giant tour. This kind of indirect search avoids having to handle facility and vehicle capacity violations resulting from the perturbation. The modified giant tour is split into a 2E-LRP solution by means of a heuristic version of the procedure introduced in (Prins 2004) (heuristic to keep running times acceptable), and LS is again performed on this solution. Deteriorating solutions are allowed; a child solution is accepted when the gap to the best solution found so far does not exceed a given percentage. Otherwise, the search is restarted from another initial solution.
The LS is performed via two VND improvement procedures. The first is applied to each new solution, whereas the second, which contains more complex neighborhoods, is applied only to solutions with a given maximum gap to the best one found so far. The tabu list stores recent solutions and is supposed to shorten the current ILS iteration when a tabu solution is created again. PR is embedded in the procedure as an intensification step, as a post-optimization step, or both. A fixed-size pool of solutions is created, into which locally optimal solutions are inserted if their objective function value is below a certain threshold. If the pool is full, a new solution replaces the oldest one. PR uses the notion of a 'big tour', which is a sequence of customers into which level-0 facilities are inserted but the tours on the first echelon are not. According to the authors, this is because generating path trajectories between two 2E-LRP solutions or between two giant tours did not work well; the first yielded too many infeasible intermediate solutions, the other was too time-consuming.
Like the one described in (Nguyen et al. 2010), this procedure requires a number of parameters to be set. Again, the authors perform thorough statistical tests to determine good setups. Computational experiments are performed with the NPP-N, NPP-P, and PPW instances as well as with the larger S instances (those with at least 50 customers). On the NPP-N and NPP-P instances, the heuristic is compared with two unpublished algorithms by the same authors and clearly outperforms these simpler procedures. On the PPW instances, which are standard LRPs, the heuristic is again compared with the procedures described in (Prins et al. 2006a,b, 2007) and (Duhamel et al. 2010). It achieves competitive results, albeit at slightly higher computation times.
Nguyen et al. (2012a) solve the variant studied in the previous two papers and describe a GRASP reinforced by a learning process and PR. A two-index arc-variable formulation is presented. Four constructive heuristics are used: the three heuristics described in (Nguyen et al.

2010) and a heuristic consisting in the construction of giant tours and applying a splitting procedure generalizing the one presented in (Prins 2004). The two VND improvement procedures described in (Nguyen et al. 2012b) are applied. The GRASP contains a diversification and an intensification mode that are iteratively executed, similar to the procedure used in (Prins et al. 2006a). The intensification procedure constitutes the learning process. The PR step follows the same ideas as in Nguyen et al. (2012b). PR is used within the GRASP each time a solution is accepted in the pool, after the GRASP is completed, or in both situations.
As in the previous two works, extensive parameter testing and evaluation with statistical methods is done. Computational experiments are performed with the NPP-N, NPP-P, and PPW instances. On the NPP-N and NPP-P instances, the complete heuristic is compared with several partial versions that comprise only one of the constructive heuristics and one of the VND searches. Not surprisingly, the complete version yields by far the best results. On the NPP-P instances (which are constructed from standard LRP instances), the results are additionally compared to an extended version of the procedure proposed in (Prins et al. 2007) for the standard LRP. Here, results are mixed: The complete heuristic finds fewer best solutions than the extended Prins et al. (2007) procedure, but it performs more stable and has a significantly smaller average gap.
Hemmelmayr et al. (2012) present an Adaptive Large Neighborhood Search (ALNS) heuristic for the 2E-VRP (facilities have no fixed opening costs and are uncapacitated) with one level-0 facility. The paper is included in this survey because the authors show how standard LRPs can be modeled and solved as a 2E-VRP. This works as follows: A dummy vertex is created for the single level-0 facility. The potential facilities of the LRP are used as level-1 facilities. The fixed costs for opening the LRP facilities are put on the links from the dummy vertex to the level-1 facilities in the 2E-VRP. No links are introduced between level-1 facilities. For each level-1 facility, there is a dedicated vehicle that can visit only its assigned facility and the dummy vertex and has a capacity equal to that of the corresponding LRP facility.
The ALNS first constructs a feasible solution by assigning each customer to its closest level-1 facility and then solving the resulting VRPs on the second and first echelon with the savings heuristic. Eight different destroy operators are used, some of which change only the assignment of customers, whereas others also allow to open or close one or more facilities. As repair operators, variants of greedy and regret insertion are used. LS is applied on the routes of both echelons with several standard move types. No LS moves affecting location decisions are performed. Infeasible solutions (violating vehicle and, for the LRP, facility capacities) are allowed and penalized in the objective function.
Extensive computational experiments are performed with the larger GPTV instances (with at least 21 customers) and the 50-customer CPMT instances for the 2E-VRP, and the TB, B, and PPW instances for the standard LRP. In addition, a set of 2E-VRP instances is created by appropriately modifying 17 PPW instances with more than 50 customers. The ALNS performs very well on the 2E-VRP and the LRP instances: In comparison with several solution approaches from the literature (Perboli et al. 2010, 2011, Crainic et al. 2010a for the 2E-VRP, Tuzun and Burke 1999, Prins et al. 2006a,b, 2007, Duhamel et al. 2010, Pirkwieser and Raidl 2010, Yu et al. 2010 for the standard LRP), it provides the best average solution quality.
Contardo et al. (2012) study the 2E-LRP with several capacitated level-0 and level-1 facilities and fixed opening costs for facilities on both levels, where routes must be computed on both echelons. They describe a branch-and-cut algorithm and an ALNS. The central observation made by the authors, exploited in both solution approaches, is that the 2E-LRP can be decomposed into two standard LRPs (one for each echelon) that are connected via the level-1 facilities.
The branch-and-cut algorithm is based on a two-index arc-variable formulation. Besides the binary arc variables, the formulation has continuous variables indicating the amount of load passing through a level-1 facility. These variables are used to connect the two LRPs at each echelon: For the first echelon, the variable values correspond to demands at the level-1 facilities, for the second echelon, the values correspond to facility capacities. This relationship allows to use most of the valid inequalities for standard LRPs introduced in the papers discussed in

Section 4.1. The authors introduce several new types of inequalities and describe separation procedures.
The ALNS heuristic recursively calls a modification of the ALNS described in (Hemmelmayr et al. 2012). An initial solution is constructed for the second echelon in a manner similar to the one described in the latter paper. Then, a solution for the first echelon is built by opening randomly one level-0 facility and serving all level-1 facilities from it. This may violate facility capacities, but, as in (Hemmelmayr et al. 2012), infeasible solutions are allowed and penalized. Given a solution, a destroy-repair iteration is first performed on the second-echelon problem. Then, the resulting first-echelon problem undergoes a destroy-repair step. LS is performed only on the second echelon. The destroy and repair operators as well as the LS moves are similar to those used in (Hemmelmayr et al. 2012).
Computational experiments are performed with the NPP-N, NPP-P, and S instances. The ALNS clearly outperforms the approaches described in Nguyen et al. (2012a) and Nguyen et al. (2012b) on the tested instances. The branch-and-cut algorithm is compared to the three-index arc-variable formulation described in Boccia et al. (2011) (which was re-implemented by Contardo et al. (2012)). The results show that the two-index formulation by Contardo et al. (2012) solves more and larger instances and provides tighter gaps for those instances that cannot be solved to optimality. A comparison of the results of the ALNS and the branch-and-cut algorithm shows an average gap of only about 3% between the upper and lower bounds provided by these procedures. This demonstrates the quality of both algorithms.
Schwengerer et al. (2012) present a VNS approach for the 2E-LRP. They extend the VNS of Pirkwieser and Raidl (2010), which was originally designed for the (P)LRP. The authors determine an initial, not necessarily feasible solution as follows. Level-1 facilities are opened randomly one by one until the capacity is sufficient to accommodate the complete customer demand. Each customer is assigned to its closest open facility. Then, routes are computed for each level-1 facility with the Clarke and Wright savings algorithm. The fleet size is limited, therefore, if the number of created routes exceeds the number of available vehicles, this number is reduced by greedily re-assigning customers from least-customer routes to other routes. The same steps are repeated for the first echelon, taking the opened level-1 facilities as customers with demands equal to their associated cumulated customer demands. Contrary to the approach used in (Pirkwieser and Raidl 2010), penalties for infeasible solutions are varied in the course of the algorithm.
In the shaking phase, six different neighborhood structures are used, and 21 shaking neighborhoods are applied overall. The basic neighborhoods are (i) exchange two segments of variable length between the routes of the same facility, (ii) as before, but between different facilities on the same echelon, (iii) close one level-1 facility and open another one, (iv) open or close one level-1 facility, (v) close one level-0 facility and open another one, (vi) open or close one level-0 facility. A fixed shaking neighborhood order is applied. As intensification, 3-opt as intra-route and 2-opt* as inter-route neighborhood are used. Moreover, deteriorating solutions are accepted according to an SA criterion.
Computational experiments are performed with the NPP-P, NPP-N, and S instances. The results show that the VNS is competitive with existing approaches (Nguyen et al. 2012a,b, Contardo et al. 2012). Although the VNS exhibits slightly higher average gaps than the ALNS of Contardo et al. (2012), it is able to identify or improve the BKS for a significantly larger number of instances.
Ambrosino et al. (2009) study a very special case of a 2E-LRP. In their problem, the set of customers is partitioned into given clusters (regions). There is one given central facility and, for each region, one local facility must be established at one of the customer locations. Each customer's demand consists of two parts. One part must be delivered from the central facility, the other from the local facility. Therefore, routes must start at the central facility and go directly to a local facility before visiting customers of the corresponding region and returning to the central facility. A fixed fleet of heterogeneous vehicles is available to this end.

The authors describe a two-stage matheuristic. In the first stage, it computes a feasible solution by solving, for each region, an IP for clustering customers into groups to be visited by one particular vehicle and for determining the facility to open in the region. For each such cluster, an asymmetric TSP is then solved with a branch-and-bound code to determine the visit sequence. In the second stage, the given solution is improved by first trying to replace a local facility with a different one. Then, the routes within each region are improved using a neighborhood based on cyclic exchanges of customers between routes. This neighborhood can be represented by a suitably defined digraph, where a cyclic exchange corresponds to a special negative cost cycle. Finding such cycles is NP-hard, but the authors use an efficient heuristic search method.
Computational experiments are performed with random self-generated instances with up to 420 customers in 6 regions and one real-world instance with 200 customers in 5 regions. For the large instances, a commercial solver fails to compute a feasible solution within 25 hours. The heuristic finds feasible solutions for all instances and optimal solutions for instances with up to 40 customers. For the large instances, the gap between the heuristic solution and a lower bound is 10% on average.

5.2 Two-echelon LRPs with temporal aspects

None of the previous papers considered temporal aspects of synchronization at the intermediate facilities, but several of them mention this as an important topic for further research. The papers in this section treat problems containing such aspects.
Burks (2006) studies the so-called theater distribution problem, which is a 2E-LRP with multiple commodities, time windows, a limited, heterogeneous fleet, and capacitated facilities. In addition to the location and assignment decisions regarding the facilities, vehicle depots have to be selected out of a discrete set of potential depots, and the available vehicles must be assigned to a depot. Direct transports from level-0 facilities to customers are allowed. In contrast to all other NE-LRPs reviewed here, the fleet is not partitioned into vehicle classes that are restricted to operate on one particular echelon. The vehicles differ with respect to costs, capacities, temporal availability and the commodities they can transport. The objective is to minimize the fixed depot and facility opening and vehicle usage costs, and the variable facility operating and distance-dependent vehicle routing costs.
The author presents an arc-variable based MIP model for the problem and a solution approach based on Adaptive TS (ATS). For the description of the solution representation and the neighborhood and move definitions, concepts and terms from algebraic group theory are used. A solution is represented with the cyclic form of the symmetric group on n letters, where n is the sum of the number of depots, facilities, customers, and vehicles. Neighborhoods are described by conjugacy classes, and moves are described using function composition and conjugation. To construct an initial solution, a simple greedy heuristic and a sequential insertion procedure are used. Three levels of solution (in)feasibility are considered: Infeasible solutions violating hard constraints such as vehicle-request compatibilities, near-feasible solutions violating time window constraints, and feasible solutions fulfilling all constraints. All infeasibilities are penalized in the objective function, and the penalties are adjusted during the solution process. The ATS uses four different insert and two different swap neighborhoods, some of which allow opening or closing depots or facilities. A dynamic move- and solution-based tabu list is maintained. Also, an elite list of the best found feasible and near-feasible solutions is kept. After a predefined number of iterations without improvement, a restart is performed with an elite solution as the new incumbent.
The computational experiments are an outstanding feature of the work. To determine suitable values for the ATS parameters, the author conducts an extensive and sophisticated statistical analysis. The assessment of the algorithm's performance is based on problem characteristics. Eleven evaluation questions are formulated (e.g., 'How do instance characteristics such as time window width affect the number of TS iterations necessary to achieve a certain solution quality?' or 'Does the use of elite lists affect the solution process?'). Overall, 16 factors in three

categories concerning problem (scheduling, e.g., time window width, and routing, e.g., number of potential facilities) as well as algorithmic aspects (e.g., tabu tenure) are considered. To determine the significance of these factors with regard to the questions posed, an advanced Design Of Experiments (DOE) approach is taken.
To conduct the experiments, random instances are generated. The two most important results are that the time window width has a significant effect on the solution structure (number of open facilities and used vehicles), and that the use of elite lists considerably improves solution quality. Furthermore, the quality of the heuristic solutions (obtained with the settings determined in the DOE experiment) is evaluated by comparing the objective function values to optimal solutions and lower bounds. Optimal solutions are obtained for small instances by solving the proposed formulation with a commercial solver. Lower bounds are computed by decomposing the problem into a location and a routing subproblem, solving the subproblems to optimality, and adding the two objective function values. For the 25 instances that could be solved to optimality, the ATS finds the optimal solution in 22 cases, and in the other three, the gap is below 2.5%. The optimal number of depots, facilities, and vehicles is determined in all cases. For the instances for which a lower bound is computed, the ATS has an average gap of 2.6%. Finally, the ATS is compared with an existing, partly manual planning software system on two large random and three real-world instances. The results show that the ATS yields significantly better results in shorter time.
Aksen and Altinkemer (2008) extend the research reported in (Özyurt and Aksen 2007, see Section 4.2) and study a variant of the 2E-LRP with given uncapacitated level-0 facilities and two different types of capacitated level-1 facilities. The authors assume existing level-1 facilities of one type, which may be closed or transformed into the other type. In addition, new facilities of either type may be opened at a fixed cost. On the first echelon, direct goods flows are determined. Routes are computed only for the second echelon. The customers have single-sided time windows (latest arrival). Not all customers need to be served: Customers have a certain maximum distance to the nearest open level-1 facility. If no facility is open within this distance, the customer demand is lost at a penalty.
The authors provide an MIP formulation for their problem. Essentially, the formulation represents the problem as a Two-Echelon Uncapacitated FLP (2E-UCFLP) and an MDVRP with time windows and links the two subproblems by means of appropriate constraints. The problem is solved by nested LR and subgradient optimization. Upper bounds are provided and improved by heuristics. In the LR, the constraints linking the two subproblems are relaxed. The relaxed problem then decomposes into a 2E-UCFLP and a capacitated minimum spanning forest problem with single-sided time windows. The former is solved to optimality in each subgradient iteration by a commercial solver. The latter is again solved by LR.
Computational experiments are performed with self-generated random instances with one level-0 facility and up to five level-1 facilities and 300 customers. The results show that, although most instances cannot be solved to optimality (i.e., a gap of more than 0.1% between the upper bound provided by the heuristics and the lower bound provided by LR remains), the method clearly outperforms a direct solution of the formulation with a standard solver.
Crainic et al. (2007) and **Crainic et al. (2009)** deal with an extended version of the 2E-LRP in the context of city logistics. A multi-commodity setting is considered, i.e., the demand of each customer consists of a particular, non-substitutable consignment that must be picked up at a designated level-0 facility and delivered to the customer through a two-echelon transport network. The authors study models where an exact synchronization in space and time of the vehicles of the two echelons is required. A first-echelon vehicle may only arrive at a level-1 facility when there are enough second-echelon vehicles to receive the complete load of the first-echelon vehicle. Vehicles must not wait, and load cannot be stored at the level-1 facilities. In the problem versions studied, using a facility incurs no fixed costs, but the level-1 facilities are capacitated in the sense that the number of vehicles of both echelons that may simultaneously use a facility is limited. Consequently, routing decisions must take into account locational aspects; the location decisions are not implicitly determined by the routing decisions.

The authors develop a model using path variables, based on a time-discrete network where there is one vertex for each pair (level-1 facility, time period). There are three types of path variables: One each for the paths/routes of the first- and second-echelon vehicles, and one for the path each customer request takes. This is necessary because the goods to be transported are not substitutive.
The work is essentially a modeling paper, so no detailed algorithmic developments are described, and no computational experiments are performed. Nevertheless, the authors propose a heuristic hierarchical decomposition by echelon. For the second-echelon model, a further heuristic decomposition into the sequential solution of a vehicle routing and a network flow component is proposed. The vehicle routing component solves, for each (facility, time period) pair, a VRP with time windows for the customers associated with the respective pair. The flow component solves a network flow problem to provide the (facility, time period) pairs with a sufficient number of vehicles. Overall, the presented models are very involved and give an impression of the difficulty of actually solving such problems.
Nikbakhsh and Zegordi (2010) study a problem version with capacitated level-0 and level-1 facilities and fixed costs for opening level-1 facilities. On the first echelon, only direct transports are considered. Routing decisions are made on the second echelon. Customers have time windows (a, b, c) with $a \leq b \leq c$. Visiting a customer before time a is impossible and requires waiting. Visiting a customer after time a and not later than time b is allowed at no penalty. Visiting a customer after time b and not later than time c is allowed at a fixed penalty. Visiting a customer after time c is infeasible. An upper bound on the number of vehicles that may be used at each level-1 facility and a maximum route duration are specified.
The authors present a three-index arc-variable MIP formulation, compute a lower bound for the problem based on this formulation, and develop a heuristic solution procedure. The lower bound is determined by relaxing a set of constraints that link the location with the routing aspects. The problem then decomposes into a FLP for the first echelon and a location-routing subproblem for the second echelon. The latter subproblem is simplified to a Degree-Constrained Capacitated Minimum Spanning Forest Problem with vehicle capacities and maximum route duration, where the time window aspect is ignored. This problem is solved in two ways: by LR (of the capacity and route duration constraints) with subgradient optimization (similar to the procedure described in (Aksen and Altinkemer 2008)), and by a heuristic extension of the Kruskal algorithm (where capacity and route duration constraints are ignored).
The heuristic is an extension of the two-stage LRP heuristic by Albareda-Sambola et al. (2007). First, an initial feasible solution for the second-echelon subproblem is constructed by opening level-1 facilities one by one based on the ratio of fixed costs to capacity. Customers are assigned to the newly opened facility and routes are built as long as the constraints on the maximum number of allowed vehicles at the facility, the vehicle capacities, and the time windows are maintained. Then, an LS is executed using 3-opt moves and the neighborhoods defined in (Albareda-Sambola et al. 2007) as well as two new ones. These neighborhoods are applied iteratively in a fixed sequence and until a stop criterion is met. The costs considered in the evaluation of a move are the fixed opening costs for level-1 facilities, routing costs on the second echelon, penalties for time window violation and the costs for the direct transports from level-0 to level-1 facilities resulting from the customer assignment on the second echelon.
Computational experiments are performed on self-generated random instances with up to 10 level-0 facilities, 50 level-1 facilities and 100 customers. The gap of the heuristic to the computed lower bound is 8.55% on average. For small instances, where optimal solutions could be computed with a standard solver, the gap is below 1.9%.

5.3 Multi-echelon LRPs with more than two echelons

There are only a few papers on systems with more than two echelons. Given the practical relevance and scientific challenge of such problems, this comes as a surprise. Prior to 2006, there is only one paper, the seminal work by Ambrosino and Scutellà (2005), which was already reviewed in (Nagy and Salhi 2007).

Gonzalez Feliu (2009) presents a path-based MIP model for the general N-echelon LRP but performs no computational experiments.

Lee et al. (2010) study a three-echelon LRP with routing decisions on the first and third echelon. They consider capacitated facilities on levels 1–3 and fixed costs for opening facilities on levels 1 and 2. Two MIP models are developed. The first one considers only direct transports on each echelon, the second one is a three-index arc-variable based formulation for the 3E-LRP. A heuristic is presented that repeats the following three steps for a given number of iterations: First, the sets of level-1 and level-2 facilities to open are determined, then, the routing problems on echelons 1 and 3 are solved, and finally, a solution to the resulting transportation problem on echelon 2 is computed. The authors do not specify how the three subproblems are solved. Computational experiments are performed with five small self-generated random instances with up to eight customers. All instances can be solved to optimality with both models using a commercial solver, with the second one leading to better solutions because of the larger feasible region (multi-stop routes on two echelons besides direct transports). The heuristic is able to find the optimal solutions in all cases. In addition, four larger test instances with 30, 10, and 10 facilities on levels 0, 1, and 2 respectively, and 30 customers are constructed. On these instances, the heuristic is compared with a simplified procedure that solves the three subproblems just described only once, using the LRP heuristic by Wu et al. (2002) for the routing problems and LP to solve the transportation problem. The heuristic clearly outperforms the simplified procedure.

Hamidi et al. (2012a), **Hamidi et al. (2012b)**, and **Hamidi et al. (2014)** study a three-echelon LRP with multiple commodities, capacitated facilities on levels 0–2, existing facilities on level 0, fixed costs for opening facilities on levels 1 and 2, and a limited number of capacitated, homogeneous vehicles with fixed as well as variable costs and a limit on the route length. Transports are allowed from any level-n location to any location on level $n' > n$, and horizontal transports on the same level are possible on levels 0 and 1. Moreover, customers may be served from a facility on any level. Routes are only allowed for deliveries to customers; transports between facilities must be direct. Hamidi et al. (2012a) present a three-index arc-variable based MIP model for this problem, and Hamidi et al. (2012b) describe a metaheuristic. The procedure decomposes the problem into two subproblems, a location-allocation-transshipment problem, and a routing problem. GRASP and TS are combined to solve the first subproblem in which the routing cost is considered through an approximation. The second subproblem is solved with a combination of the savings heuristic and a node ejection chain procedure. Hamidi et al. (2014) present an extended and improved version of the heuristic. Computational experiments are performed with self-generated random instances with 3, 20, and 30 potential facilities on levels 0, 1, and 2, 380 customers, and 5 commodities.

6 Periodic and multi-period LRPs

This section first describes the works on Periodic LRPs (PLRPs) and then one reference we found on a multi-period LRP that does not fall into the latter category. PLRPs combine the standard LRP with the Periodic VRP (PVRP, see, e.g., Francis et al. 2008), in which trips have to be planned over a multi-period horizon. The periods of visit for each customer can be selected from a set of allowed visiting patterns. PLRPs aim to determine (i) the facility configuration used in all periods, (ii) the assignment of visiting patterns to customers, (iii) the assignment of customers to facilities for each period of the planning horizon (a customer is not necessarily assigned to the same facility in each period), and (iv) the vehicle routes of all facilities for all periods. The objective is to minimize the sum of facility opening costs, fixed vehicle costs, and routing costs. Note that in the PLRP, vehicles are assumed to be stationed at facilities over all periods. Therefore, the vehicle fixed costs for each facility are based on the maximal number of vehicle routes that are performed from this facility in any period of the horizon.

Prodhon (2008) presents an iterative three-stage heuristic for the PLRP. In the first stage, the PLRP is transformed into a single-period LRP, where all customers of the multi-period horizon have to be served in one period, with their demands and facility capacities adjusted accordingly. A set of solutions to the resulting problem is generated with several iterations of the RECWA of Prins et al. (2006a) run in diversification mode. The best facility configuration found is used for the current global iteration of the algorithm. In the second stage, a parallel insertion heuristic assigns visiting patterns to customers based on the frequency of edges in the set of solutions generated in the first stage. The procedure is based on the idea that customers that are consecutive in many LRP solutions of the first stage are also likely to be consecutive customers in a PLRP solution, if this is compatible with the visiting patterns. Therefore, the edges are sorted in decreasing order of their frequency and are then selected with a given probability starting from the most frequent edge. The visiting pattern is selected in a way that inserts the edge at minimum cost in a maximum number of periods (the details of the procedure are not described in the paper). Afterwards, the obtained solution is improved separately for each period by the LS of Prins et al. (2006a).
In the third stage, for each period, an MDVRP is solved with the RECWA run in intensification mode, followed by LS. In addition, two LS steps that consider the entire planning horizon are applied. The first one tries to find assignments of patterns to customers that lead to reduced routing costs by exchanging the visiting patterns of customers, and the second one aims at reducing the number of vehicles assigned to a facility over the periods. The algorithm finishes after a given number of global iterations of the three stages.
Tests are conducted on the Prodhon instances for the PLRP (introduced in this work), on the PPW instances for the standard LRP, and the PVRP instances of Cordeau et al. (1997), where the latter two problems represent special cases of the PLRP. On LRP and PLRP a decent solution quality is obtained (approximately 2% gaps to (i) the BKS at the time for the standard LRP, and (ii) the solutions obtained by tripling the number of iterations for the PLRP), whereas on the PVRP instances rather significant gaps to the BKS (around 6%) can be observed.
Prodhon and Prins (2008) adapt MA|PM to the PLRP. All individuals in the population of one generation are assumed to have the same assignment of visiting patterns to customers, and these patterns are not changed in the genetic process. As a consequence, chromosomes of the same length can be generated by representing a PLRP solution as a concatenation of LRP solutions for each period of the planning horizon, where the single-period solutions are encoded as described in (Prins et al. 2006b, see Section 4.3.1). Note that the facility configuration of all parts of the chromosome must be identical since the same facilities are open during all periods of the planning horizon. Decoding of a chromosome works as described in (Prins et al. 2006b), with the one difference that the splitting of the giant tours of the facilities must be performed for each period.
The algorithm uses binary tournament to select parents and performs the crossover described in (Prins et al. 2006b) separately for each chromosome part corresponding to a single period. The crossover is followed by an extended version of the repair procedure described in (Prins et al. 2006b) and an LS that is similar to the one described in (Prins et al. 2006a) and executed with a certain probability. Contrary to the original work, the distance measure used in the population management step is not based on structural properties of the solutions but on their fitness values. After the crossover and LS, an additional LS step on the visiting patterns is performed. If the resulting solution is superior to all solutions in the current population, its assignment of visiting patterns is recorded to diversify the search in later generations. Note that the solution found in this LS step is not included in the population because all individuals must have the same visiting patterns. The algorithm stops as soon as a prespecified number of generations is reached.
In computational experiments on the Prodhon instances for the PLRP, the proposed method is able to improve on the solution quality of Prodhon (2008) by approximately 2% within clearly reduced run-time. However, it should be noted that strong improvements mainly occur on the small to medium-sized instances of the test set, whereas on the larger instances, the former

heuristic achieves better results than the proposed MA|PM. On the PPW instances for the standard LRP and the PVRP instances of Cordeau et al. (1997), the MA|PM is not able to match the solution quality and run-time of Prodhon (2008).

Pirkwieser and Raidl (2010) present a matheuristic based on VNS coupled with IP-based very large-scale neighborhoods for a static and a periodic LRP. The VNS uses a greedy randomized construction heuristic (choose random visiting patterns and facilities to be opened, greedily assign customers) and allows and penalizes intermediate infeasible solutions violating vehicle and facility capacity constraints during the search. In the shaking phase, five different neighborhood structures are used, each with several moves of increasing perturbation size, and 18 shaking neighborhoods are applied overall.

Three MIP-based very large-scale neighborhood searches (VLNS, see Ahuja et al. 2002) are performed, using a standard MIP solver. The first one, V1, operates on the routes of a given VNS solution and consists in solving a path-variable based (P)LRP with one variable for each route of the current VNS solution, where it is possible to open and/or close facilities and assign routes to different facilities. The second one, V2, is a set-covering model based on the model for V1 and introduces additional binary variables indicating for each customer whether or not a certain visiting pattern is chosen. The third one, V3, operates on the customer level and extracts sequences of customers from existing routes, reconnects the disconnected route parts, and then finds an optimal allocation of the extracted sequences to the possible insertion points, i.e., between any two (remaining) consecutive customers. The customers to be extracted are selected by randomly grouping customers into equal-sized subsets. V3 is performed for each such subset.

Computational experiments are performed with the PPW instances for the standard LRP and the Prodhon instances for the PLRP. The results show that, in general, the more VLNS neighborhoods are used, the better the solution is. When comparing the obtained objective function values with the BKS as reported on Prodhon's website (`prodhonc.free.fr/homepage`), Pirkwieser and Raidl (2010) find 15 (out of 30) new best solutions for the PPW and 29 (out of 30) for the Prodhon instances.

Prodhon (2011) presents an MIP formulation for the PLRP and proposes a hybrid of ELS and the RECWA of Prins et al. (2006b). An individual is represented as a list of visiting patterns for the customers, i.e., no information about the facility configuration, the assignment of customers to facilities and the routes is encoded. The fitness of an individual is evaluated by creating a PLRP solution for the list of visiting patterns using the RECWA and LS of Prins et al. (2006b). More precisely, the RECWA is first run in diversification mode for each single period of the planning horizon. As different facilities may be opened for each period, a straightforward adoption of the single-period solutions is likely to lead to a low-quality facility configuration over the planning horizon because potentially far too many facilities are open. Therefore, the overall facility configuration is derived as a subset of all facilities opened for the single periods based on (i) the maximal utilization of a facility in any period and (ii) the contribution of the facility to satisfying the total customer demand. Next, the RECWA is run in its intensification mode using the determined overall facility configuration. The results are improved by the LS of Prins et al. (2006b) on the routing, executed for each period of the planning horizon.

The overall algorithm starts with a randomly generated solution that is evaluated in the described manner and is further improved by an LS on the visiting patterns. The resulting solution becomes the starting solution for the ELS component. The latter generates a number of children by randomly mutating the visiting patterns of a given percentage of customers. The algorithm evaluates the fitness of the children, again performs the LS on visiting patterns, and the best child becomes the starting solution of the next ELS iteration if it improves on the previous starting solution. After a certain number of ELS iterations is reached, a new individual is randomly generated for the next global iteration of the algorithm. The overall algorithm stops after a fixed number of total solution evaluations.

On the Prodhon instances for the PLRP, the hybrid improves on the results of the MA|PM of Prodhon and Prins (2008) by more than 7% considering the average solution quality and by more than 9% considering the best solution, while requiring roughly the same amount of run-time. On the PPW benchmark for the standard LRP, a slightly improved performance compared to MA|PM can be witnessed. The required run-time is on a level with that of MA|PM but clearly slower than the dedicated methods described in (Prins et al. 2006a,b, 2007). Finally, an average gap of 2.6% percent to the current BKS of the PVRP instances of Cordeau et al. (1997) is obtained and the solutions of MA|PM can only be slightly improved on these instances.

Prodhon (2009a) presents a preliminary version of the ELS-RECWA hybrid of Prodhon (2011) that features a PR component. Contrary to (Prins et al. 2006a), where PR is used as post-optimization step, PR is applied as intensification step between the best solution found at the end of an ELS iteration and the most distant solution in an elite set composed of the best solutions at the end of previous ELS steps. Here, the distance is defined as the number of customers with different visiting patterns in the two solutions. Transformation takes place by replacing the visiting patterns of the original solution with those of the guiding solution and is followed by the evaluation step with RECWA and the LS on visiting patterns described above. Computational experiments on the Prodhon instances for the PLRP show that the proposed algorithm is clearly able to improve on the solution quality of Prodhon and Prins (2008) and that of another memetic algorithm presented in (Prodhon 2009b). Although results are not directly comparable with those of Prodhon (2011) (the algorithm uses a different parameter setting and a different number of test runs than Prodhon (2011)), it seems that the presented version with PR is not able to match the solution quality of the method presented in Prodhon (2011), but it has lower run-times.

Albareda-Sambola et al. (2012) study a multi-period LRP with uncapacitated facilities and vehicles and different time scales for the location and for the routing decisions. This means that routing decisions are made in each period, whereas decisions on opening of facilities are made only in certain prespecified periods. The problem is not a periodic LRP because the customer demand in each period is specified in advance, so that there is only one visiting pattern per customer.

The authors provide an arc-variable based MIP model. To solve the problem, a relaxation is considered where routing decisions are approximated by forests rooted at available facilities. This relaxation is considerably easier to solve with standard software than the original problem. After solving the approximation, solutions to the original problem are obtained by optimally solving a series of TSPs for each time period.

Computational experiments are performed with self-generated random instances with up to 20 facilities, 70 customers, 12 time periods for routing, and 4 time periods for location decisions. The results show that, for instances that could be solved to optimality, the solution to the relaxation usually provides excellent approximations to the original problem, both in terms of the facilities to open at the different time periods and the customers to be served by each of the available facilities.

7 LRPs with pickup and delivery

In this section, we first describe two articles on the LRP with Simultaneous Pickup and Delivery (LRPSPD) and subsequently two articles on the many-to-many LRP (MMLRP).

Karaoglan et al. (2011) present a branch-and-cut algorithm for the LRPSPD. They consider a directed graph with homogeneous fleet and develop a formulation with five types of variables: Binary variables indicate whether or not an arc is traversed, whether or not a facility is opened, and whether or not a customer is assigned to a facility. Continuous variables indicate the demand to be delivered to customers routed after vertex i and transported over arc (i, j) if a vehicle uses that arc, and the demand to be picked up from customers routed up to i and transported over (i, j). The authors describe several types of valid inequalities. Upper bounds are computed in two ways: (i) A feasible initial solution is computed with the RECWA by Prins et al. (2006a). (ii) In the course of the algorithm, further feasible solutions are determined from fractional solutions

by a greedy rounding heuristic. The solutions are improved by an SA metaheuristic using four different LS neighborhoods.
Computational experiments are performed with the KAKD instances, which are introduced in this paper. The algorithm is compared to its simplified version without upper bounding and to the direct solution of the instances with an MIP solver; it clearly outperforms these two alternatives.
Karaoglan et al. (2012) present an alternative LRPSPD formulation without the continuous flow variables. Instead, variables indicating the delivery and the pickup load just before and just after having served a customer are used. Computational experiments are performed with the KAKD instances described in (Karaoglan et al. 2011). The two formulations are compared with respect to which instances are solved to optimality and with respect to the quality of the lower bounds obtained. The result is that no formulation strictly outperforms the other. Moreover, an iterative heuristic is proposed. First, a feasible initial solution is computed with one of two approaches: either with the RECWA, as in (Karaoglan et al. 2011), or by solving the location subproblem as an SSCFLP with a standard solver and then the routing subproblem with the routing part of the RECWA. In both cases, the result is improved with SA as in (Karaoglan et al. 2011). A location phase follows where three move types are considered (add, drop, swap). The procedures for routing and location improvement are repeated until a stopping criterion is met. Computational experiments show that better results are obtained with the heuristic if the second approach is used.
Çetiner et al. (2010) consider an MMLRP application arising in postal logistics and make the following assumptions: For each pair of customers, a required flow of goods in either direction is specified. Hubs are uncapacitated, and vehicles have no loading constraints, but a maximum route length is specified. The pickup and the delivery at a customer are performed simultaneously. Each customer may be assigned to more than one hub, i.e., may send its outgoing goods to several hubs and receive its incoming goods from several hubs. The objective is hierarchical: First, minimize the number of vehicles used subject to a specified maximal direct distance between a customer and any of its assigned hubs, and then minimize overall transport costs. There are no fixed hub or vehicle costs, but a prespecified number p of hubs must be used.
To solve the problem, a nested iterative two-stage matheuristic is used. In the first stage, a multiple allocation p-hub median problem is solved, while in the second stage, multiple TSPs with restricted tour length are tackled for each hub opened in the first stage. The second-stage problems are first solved for a given upper bound on the number of vehicles at each hub, and then repeatedly solved with a decreasing number of vehicles until no feasible solution is found, thus minimizing the number of vehicles. For both problems, an exact solution is computed using MIP models from the literature and a standard MIP solver. After each iteration, the distances between customers and hubs in the first-stage problem are updated using the results of the second stage. The process is repeated until the solution no longer changes between iterations.
Computational experiments are performed with seven modified benchmark instances taken from four different sources belonging to the facility location literature and one real-world instance, with up to 81 customers. The results obtained with the solution procedure are compared to those obtained when only one iteration of the procedure is performed, and savings of more than 20% are reported.
de Camargo et al. (2013) also study an MMLRP. The assumptions underlying their problem are similar to those of Çetiner et al. (2010), but de Camargo et al. (2013) assume that (i) each customer is assigned to exactly one hub (single assignment), so that each customer is visited exactly once, and (ii) each customer location is considered a potential hub location. The objective is to minimize the sum of fixed costs of installing hubs, handling costs incurred for transferring goods at hubs, fixed costs for assigning vehicles to open hubs and distance-dependent costs for the local vehicle routes and the inter-hub transports. The authors propose an arc-variable based MIP model combining the model of Skorin-Kapov et al. (1996) for the single allocation hub location problem with the model of Claus (1984) for the TSP. These models are used because of the strength of the respective linear relaxation.

The problem is solved to optimality by Benders decomposition embedded in a branch-and-cut framework. The Benders subproblem is further decomposed into two different problems, the first one a transportation problem, the second one a pure feasibility problem. These two sub-subproblems can again be decomposed: The first one into transportation problems between single pairs of customers; the second one into feasibility problems to decide whether the current master problem solution contains a feasible routing for a certain vehicle assigned to a certain hub. By this decomposition of the subproblem, it is possible to generate both optimality and feasibility cuts simultaneously in one iteration of the Benders decomposition algorithm. The authors refine the algorithm and accelerate its convergence by adding only Pareto-optimal cuts (Magnanti and Wong 1981, Papadakos 2008) and by using a special cut selection technique (Fischetti et al. 2010) for choosing the feasibility cuts to add.
For computational experiments, test instances with 10–100 customers are generated from a data set for hub location problems. For instances with up to 30 customers, the authors compare their algorithm with a direct solution of the formulation by a standard MIP solver and observe an acceleration factor of between 2 and 100. The largest instance solved to optimality with the Benders decomposition algorithm has 100 customers. This is remarkable, as a 100-customer-instance corresponds to 10,000 commodities and contains more than four million integer variables.

8 Stochastic and fuzzy LRPs

The papers in this section consider nondeterministic data for one or more problem aspects, such as customer demands or travel times. First, stochastic LRPs assuming a known probability distribution are discussed, then fuzzy LRPs where the uncertain data is given in the form of fuzzy numbers.
Ahmadi-Javid and Seddighi (2013) study the following stochastic LRP: During a planning horizon, customers should be visited several times, and each visit should be performed by the same vehicle along the same route starting at the same facility. The capacity of a facility and the number of times a vehicle can perform its assigned route during the horizon are modeled as discrete random variables with finite support. The stochastic objective is to minimize the sum of (i) fixed facility opening costs, (ii) variable routing costs, and penalty costs if (iii) the realization of a facility capacity is below the sum of demands of the assigned customers, or (iv) the realization of a random variable modeling the number of times a vehicle can perform its route is below a desired value.
The authors develop a three-index arc-variable MIP for the problem and use a simplified version of the metaheuristic described in Ahmadi-Javid and Azad (2010) (see Section 10). To deal with the stochastic objective, they propose a moderate, a cautious, and a pessimistic risk-management policy to scalarize its stochastic components. The authors provide a discussion of risk measurements and policies as well as theoretical analyses on the potential impact of the different policies on solution quality.
Computational experiments are performed with self-generated random instances with 2–30 facilities and 4–200 customers. Instances with up to 4 facilities and 9 customers can be solved to optimality with a commercial solver. Using a relaxation of the MIP model, lower bounds are computed for medium-sized instances with 8–17 facilities and 20–60 customers. The heuristic computes solutions that are at most 7% above the lower bounds. Scenario analyses shows that for any risk measure, the obtained results improve significantly compared to the case where the stochastic components of the objective function are ignored.
Other authors that study stochastic aspects are **Hassan-Pour et al. (2009)**, who consider a multi-objective problem where the availability of facilities and transport links is stochastic, and **Zhang et al. (2008)** and **Ahmadi-Javid and Azad (2010)**, who consider stochastic inventory LRPs. These works are reviewed in the sections on multi-objective and on inventory LRPs (Sections 9 and 10) respectively.
Zarandi et al. (2011) address an LRP with capacitated facilities and vehicles and time window constraints. Travel times are uncertain and represented as triangular fuzzy variables. The

problem is modeled according to the credibility theory of Liu (2004), and the authors present an SA algorithm. Time windows are not treated explicitly in the algorithm and the time windows of the one test instance investigated in the numerical experiment are the same for all customers and all facilities. To assess the quality of the algorithm, it is run on one 20-customer instance of the PPW benchmark set for the standard LRP and is able to find the BKS for this instance.

Zarandi et al. (2013) study an extension of the problem with fuzzy customer demands. The proposed SA algorithm uses a different initialization procedure and treats the time window aspect by discarding initial solutions that violate time windows. The generated test instances again have identical time windows for all customers. Numerical studies investigate the influence of different problem parameters on self-generated instances with 100 customers and 5 potential facilities. Moreover, a random selection of 9 small instances of the B and PPW sets for the standard LRP are solved. The proposed approach shows deviations between 0% and 4.4% to the BKS.

Zare Mehrjerdi and Nadizadeh (2013) study an LRP with capacitated facilities and vehicles and uncertain demands given as triangular fuzzy variables, and which is modeled based on the credibility theory of Liu (2004). To solve the problem, the clustering heuristic of Sahraeian and Nadizadeh (2009) is adapted to the fuzzy problem and stochastic simulation is used to determine the actual demands of the customers. Numerical experiments are carried out on three self-generated problem instances. The authors investigate the optimal risk attitude of a vehicle dispatcher who has to decide whether to use the entire vehicle capacity and risk a so-called route failure, i.e., the vehicle has to return to its assigned facility for a refill in order to be able to serve the actual demand of the next customer. The results of the proposed method are compared to the solution obtained by a commercial solver on the relaxed problem without facility and vehicle capacities. Here, gaps between 40% and 95% are witnessed. The run-times of the proposed algorithm are rather high.

Golozari et al. (2013) present an LRP with maximum route duration constraints, where customer demands, traveling times and service times are fuzzy. The resulting model is converted into an LP by means of a fuzzy ranking function. The authors present an SA method enhanced by a mutation operator for stronger diversification to solve the problem and conduct numerical tests on small self-generated random instances.

9 Multi-objective LRPs

This section reviews papers that simultaneously deal with more than a single objective. The presented works consider a wide range of monetary and non-monetary objectives.

Lin and Kwok (2006) study an LRP with three objectives: minimize (i) the fixed facility setup costs and the variable vehicle routing costs, (ii) the workload imbalance with respect to time, and (iii) the workload imbalance with respect to load. An iterative three-stage procedure is developed and alternatively embedded in a TS and an SA metaheuristic. The first stage deals with the location aspect: The minimal number n of necessary facilities is determined by dividing total demand by facility capacity. The facilities are sorted in nondecreasing order of total distance to all customers. In each iteration of the three-stage procedure, n facilities are selected systematically from the sorted list of facilities using a tree search. The second stage is concerned with the routing aspect: A multi-depot VRP (with heterogeneous fleet, where the vehicles differ with respect to their depot and thus with respect to the distances from their depot to the customers) is solved using two versions of a savings and a nearest neighbor heuristic, followed by LS. Intra- as well as inter-route improvement steps are performed. To allow multiple use of vehicles, the third stage solves a bin-packing problem (exactly or heuristically, depending on the instance size) for each facility, where the bins are the vehicles, the bin capacity is the maximal working time of the vehicle's driver, the items are the routes, and the item size is the route duration. Two approaches are used for the routing. Either the second and third stage are performed sequentially as described or in a simultaneous fashion (no detailed description of this variant is given in the work). The routing stage takes into account only the cost objective. The values of the other two objectives are stored for each feasible solution. The solutions are then evaluated with regard to all three objectives, and efficient ones are stored to gradually build an approximation to the efficient frontier, which is steadily updated and refined in the course of the algorithm.

Computational experiments are performed with self-generated and real-world instances with 10–20 facilities and 100–200 customers. On average, the TS approach yields better results than the SA version, and the simultaneous execution of the second and third stage is superior to the sequential one on both instance types. To compare the two heuristics based on the multi-objective solutions obtained, a new and nontrivial statistical procedure, the so-called coverage measure, is proposed: 'Coverage … refers to the ability of an algorithm to generate efficient solutions spanning a wider range of values than another algorithm for each separate objective. The more wide-spread the solutions are, the more flexibility is offered to the decision-maker' (p. 1840).

Caballero et al. (2007) consider a five-objective LRP with uncapacitated facilities and describe an application concerning the installation of waste incineration facilities in southern Spain. The objectives are the minimization of (i) the fixed facility setup costs, (ii) the vehicle routing costs, (iii) the degree of rejection of a facility by towns that vehicles pass through when traveling to or from the facility, (iv) the maximal degree of social rejection corresponding to the town most affected by waste transportation, and (v) the degree of social rejection by towns close to open facilities. An upper bound on the number of facilities to be opened and route duration constraints are specified.

To solve the problem, a multi-objective metaheuristic using an Adaptive Memory Procedure (AMP) is used. The heuristic exploits the well-known fact that within a certain neighborhood of an efficient solution, another efficient solution can be found. The heuristic first generates an initial subset of the set of efficient solutions and then tries to obtain a good approximation of the complete efficient set by means of an intensification process. To generate the initial subset, a first solution is obtained with a greedy procedure that considers only the routing costs. Then, neighboring solutions are determined by performing a fixed number of AMP iterations using auxiliary objective functions and six LS neighborhoods, three of which consider only the routing aspect, whereas the others allow opening and closing facilities. All solutions found during the AMP are checked for inclusion in the initial set of efficient solutions. The intensification consists in trying to find additional efficient solutions by applying the AMP to each solution in the initial efficient set.

The quality of the approximation of the efficient set is assessed with three different measures widely used in the literature on multi-objective combinatorial optimization. A real-world instance is solved with the algorithm, but no information is given on the implementation of the solution in practice. Lacking benchmark instances for the problem type studied, computational experiments are also performed with the ADF instances (which are single-objective problems). Compared to the results reported in (Albareda-Sambola et al. 2005), where these instances are originally introduced, for nine out of 15 instances, a better solution is found, and the computation times are generally shorter.

Tavakkoli-Moghaddam et al. (2010) study a bi-objective LRP with optional customers. The first objective is to minimize the sum of fixed facility setup costs, variable facility throughput costs, and vehicle routing costs. The second is to maximize the total customer demand served. The authors present an arc-variable based MIP model and describe two metaheuristics, a Multi-Objective Scatter Search (MOSS) and an Elite TS (ETS).

The MOSS works as follows: First, a lower bound on the number of facilities, n_f, is determined by dividing the overall customer demand by the capacity of a facility. For this number, all potential facility combinations are enumerated. For each of these facility combinations, a VRP that considers only the cost objective function is solved by first assigning customers to their closest open facilities and then using the savings algorithm and considering only the cost objective function. The solution is improved by 2-opt. Next, the SS is performed as described below. After all these combinations have been examined, the algorithm tests whether the cost of the best solution is less than or equal to the cost of opening $n_f + 1$ facilities. If this is the case, the algorithm terminates; otherwise, the procedure is repeated with sets of $n_f + 1$ facilities. Efficient solutions are stored in an archive set of limited size. To avoid having to remove solutions that are important for approximating the set of efficient solutions, a solution is only added if it is sufficiently different from the solutions in the archive set.

The SS maintains a reference set (the current population from which new solutions are generated) that consists of the archive set and a set containing diverse solutions. New solutions are added to the latter set if they satisfy a distance/diversity measure. In the SS, solutions from the reference set are combined by the so-called freak path algorithm, which is similar to PR and uses a crossover procedure (that is not described in detail in the paper) to combine two solutions. An ideal point in the solution space (optimum of both objective functions) is determined and updated dynamically in the course of the algorithm. The solution from the reference set with the minimal difference to the ideal point is used as starting solution, and all other solutions in the reference set are used as target solutions. Each new solution is locally optimized with respect to the cost objective by a swap move of two customers between two routes and a relocate move.

The ETS uses the same iterative concept as the MOSS and the same facility selection procedure. A random routing solution is created and improved by the swap and relocate moves described above. An aspiration criterion is defined using a special distance measure. Overall, the ETS approach appears to be less sophisticated than the MOSS.

Computational experiments are performed with self-generated random instances. The solutions produced by the two heuristics are compared via three quality measures from the literature. The results show that the MOSS clearly outperforms the ETS.

Hassan-Pour et al. (2009) study a bi-objective LRP with uncapacitated facilities where the availability of facilities and transport links is stochastic. In other words, open facilities may fail to provide service and transport links may be unusable with a given probability. The objectives are to (i) minimize the total costs, consisting of fixed facility opening and variable vehicle routing costs, and (ii) maximize the number of customers served.

The problem is solved with a two-stage matheuristic. In the first stage, a single-objective (cost minimization) chance-constrained SSCFLP is solved exactly with a standard solver. The chance constraint is that the probability that a customer is served must be greater than or equal to a specified value. In the second stage, a bi-objective MDVRP is solved, where the depots are the facilities opened in the solution to the SSCFLP. The first objective is cost minimization, the second is minimization of the probability that a customer is not served. To solve the MDVRP, the two objective functions are scalarized, i.e., aggregated into a single objective by a weighted sum of the normalized values of each objective (normalization is necessary as the objectives have different units). The resulting single-objective MDVRP is solved with an SA that uses a shift (one customer from one route to another) operator and 2-opt in the LS.

Computational experiments are performed on self-generated random instances with up to 16 facilities and 100 customers. For small instances, a lower bound is computed by solving a relaxation of an MIP formulation for the MDVRP with a standard solver, and summing up the optimal objective function values of this relaxation and the SSCFLP. The heuristic is capable of determining solutions with an average gap of 20% to the lower bound of the cost objective within a few seconds.

10 Inventory LRPs

This section discusses works that consider the integration of inventory, location, and routing decisions into one problem.

Zhang et al. (2008) study a single-product, multi-period, stochastic Inventory LRP (ILRP). The objective is to minimize the sum of facility opening, inventory and vehicle routing costs. The customer demands for the product follow a Poisson distribution. Inventory decisions must be taken at the facilities. Customers have to be assigned to one facility for the complete planning horizon and must place an order at that facility once every period. The facilities, in turn, have

a facility-specific lead time and plan their stock levels taking into account fixed ordering and variable holding and shortage costs.
The authors present an arc-variable based MIP model and a GA. The GA uses a fixed-length binary encoding specifying which facilities are open. The fitness value of an individual equals the total cost of a solution. The facility opening costs are obvious given the encoding scheme. To compute the inventory costs, it is assumed that the demand of each customer in each period equals the expected value and that each customer is assigned to the closest open facility. Under these assumptions, the authors develop a formula for the total inventory costs in each period. Vehicle routes for each opened facility are determined with the savings algorithm. Individuals are selected for reproduction with a roulette-wheel procedure. A two-point crossover operator and a mutation operator that randomly exchanges two values in the encoding bit-string are used. The algorithm is verified using one random test instance.
Ahmadi-Javid and Azad (2010) extend the inventory-location model of Shen and Qi (2007) by transportation decisions, resulting in a static ILRP with stochastic, normally distributed customer demands and capacitated facilities with a choice of different capacity levels. Note that the problem addressed by Shen and Qi (2007) is not an LRP according to our definition because no vehicle routes are determined. Ahmadi-Javid and Azad (2010) assume a homogeneous vehicle fleet, an order size/reorder point (Q, r) inventory policy, and safety stocks at facilities. The optimal inventory policy, i.e., how often to reorder and what safety stock to keep at the facilities, considering fixed costs for placing orders and inventory holding costs at facilities, must be determined in addition to the classical LRP objectives of finding the optimal facility configuration and routing solution. The precise objective is a weighted function of location costs, costs for ordering and holding inventory and safety stocks plus transportation costs from a supplier to the facilities and from the facilities to customers. The problem is modeled as a mixed integer convex program. No stochastic optimization technique is applied. Instead, the expected value and the standard deviation of demand are used to determine expected inventory and safety stock costs.
The authors propose a metaheuristic hybrid to solve the problem. Starting from a randomly generated initial solution, location/allocation and routing decisions are iteratively tackled in two separate stages by means of a hybrid of SA and TS that is adapted to each stage by using different search neighborhoods. Vehicle routes after moves are determined by a nearest neighbor algorithm.
Studies on randomly generated instances of small to medium size show that the heuristic yields better solution quality than a commercial solver. For instances of all sizes, the method shows a stable and moderate gap to the exact solution of the relaxed problem without subtour elimination constraints. Finally, the authors show that their integrated approach is able to clearly improve on a sequential approach based on the work of Shen and Qi (2007).
Ahmadi-Javid and Seddighi (2012) consider a similar model with deterministic demands and multiple suppliers. The authors present a three-stage metaheuristic. It starts with an initial solution generated by a greedy heuristic similar to the one described in (Yu et al. 2010). In the first two stages, an SA algorithm similar to the hybrid in (Ahmadi-Javid and Azad 2010) is used to improve location/allocation and routing decisions. In the third stage, the routing is improved by a savings-based ACO. The algorithm is tested on a selection of the B instances for the standard LRP and compared to known lower bounds; however, no comparison to the results of other heuristics is made. On randomly-generated instances of the investigated problem, a strong improvement in comparison to an extended version of the method proposed in (Ahmadi-Javid and Azad 2010) is found. Finally, significant cost savings in comparison to a sequential approach considering separately (i) location and routing and (ii) inventory are reported.
Guerrero et al. (2013) study a deterministic, multi-period ILRP considering inventory decisions at both facilities and customers (retailers). The authors assume a single supplier, storage-capacitated facilities and customers, and a homogeneous, unlimited fleet of capacitated vehicles with fixed costs. In addition to the standard LRP decisions on facility configuration, customer assignment and vehicle routing, the product quantities to ship (i) from the supplier to facilities

and (ii) from facilities to retailers have to be determined. The goal is to minimize the sum of facility opening costs, transportation costs, ordering costs and inventory costs at facilities and customers. The authors present an MIP model and two sets of valid inequalities to strengthen it.
An iterative matheuristic is presented, whose general idea is to address subproblems of the ILRP and exchange information between the different solution components. In a first step, facility configuration, customer assignment, and inventory decisions are addressed by solving a Supply Chain Design Problem (SCDP) based on estimated distribution costs that is solved by means of a commercial solver. The estimates of the distribution costs are updated every time feasible routes are computed in later stages. In this way, information between the routing solution and the supply chain design is exchanged. In the next step, customer assignment and routing decisions are considered. Based on the LRP solutions for all periods, determined separately for each period by means of the RECWA of Prins et al. (2006a), each customer is assigned to a facility, and then routes for each period and facility are built with the RECWA. With some probability, the resulting solution is improved by a VND-embedded LS component addressing routing, inventory and customer assignment decisions.
The authors further propose an intensification phase to investigate the inventory and routing decision. Based on fixed locations and customer assignments, they iteratively solve a Dynamic Lot-Sizing Problem with an MIP solver and determine corresponding routes with the RECWA and the described LS. Finally, the algorithm uses a post-optimization procedure that aims at improving customer assignment and routing decisions with an ILS that perturbs the solution by randomly modifying the assignment of a given percentage of customers and then improves the perturbed solution by means of the LS.
For the computational experiments, the authors present 20 randomly generated instances with up to 5 facilities, 15 customers and 7 periods. Results are compared to a commercial solver and a sequential heuristic solving the SCDP and using the ILS to make inventory and routing decisions. On the small and medium-sized instances, the proposed method is able to improve the best solution found by the commercial solver within a time limit of 8200 seconds by 0.5% on average, while using significantly lower computational effort. The method is also able to clearly improve the solution quality of the sequential heuristic, however, with higher run-times. On the large instances with 15 customers, a considerable improvement on the best solution found by the solver in 9 hours can be observed. Here, the solution quality is again superior to that of the sequential method, but run-times are higher. Moreover, the method shows good performance on the PPW standard LRP instances and acceptable performance on the Inventory Routing Problem instances of Bertazzi et al. (2002).

11 Other LRP variants

In this section, papers on rarely-studied LRP variants or problems that could not be subsumed under one of the above section headings are discussed. We have two papers that deal with planar LRPs (Schwardt and Fischer 2009, Manzour-al-Ajdad et al. 2012), one paper on location arc-routing (Hashemi Doulabi and Seifi 2013), one that considers an LRP with outsourcing options (Stenger et al. 2012), one that studies a prize-collecting LRP (Ahn et al. 2012), one concerned with generalized LRPs (Glicksman and Penn 2008), one that examines generalized, prize-collecting, and split delivery LRPs (Harks et al. 2013), and a case study (Schittekat and Sörensen 2009).
Schwardt and Fischer (2009) study a planar LRP with Euclidean distances where a single, uncapacitated facility is to be located in order to minimize transportation costs. Vehicles are limited, capacitated and homogeneous and do not incur fixed costs. The authors extend the preliminary results published in (Schwardt and Dethloff 2005) and propose a neural network approach based on a self-organizing map as heuristic construction procedure. For each vehicle, a neuron ring is defined and the rings are connected in a central point, which is the neuron representing the unique open facility. The number of neuron rings (vehicles) must be predefined

and is initially set to the minimal possible value. It is increased in the next runs if no feasible solution can be found. The algorithm's allocation of customers to neurons determines the resulting tours, and the weight vector associated with the central point defines the location of the facility.

In order to avoid tours that are infeasible with respect to vehicle capacity, the algorithm (i) integrates vehicle capacity utilization in the distance computation for the input and weight vector, (ii) removes customers from neuron rings violating the capacity constraints, and (iii) uses a tabu counter to ensure that the removed customers are allocated to different neuron rings and are not removed from their new rings in the next iterations. However, this does not guarantee the feasibility of the final solution of a run. As a consequence, the authors conduct multiple runs for each instance. Finally, the location of the facility is improved by using the end-points of the generated tours as input to a Weber problem and solving it by means of the Weiszfeld method (Weiszfeld 1937).

Numerical tests are conducted on the VRP instances of Christofides and Eilon (1969), Gillett and Johnson (1976), Christofides et al. (1979), and Fisher (1994), which feature between 21 and 249 customers. To assess the quality of the algorithm, a set of sequential methods based on weighted and unweighted Weber problems and different savings algorithms are used for comparison. The authors show that their self-organizing map approach outperforms all comparison methods. They further point out that the parameter setting of the method is not trivial and strongly influences the success on each instance.

Manzour-al-Ajdad et al. (2012) study the same problem and also propose a heuristic solution method. The initial location of the facility is determined by means of the Weiszfeld algorithm, and new candidate facility locations are generated within an ellipsoid using the initial location as center. For each candidate, routes are built using a savings algorithm and are then further improved by an LS with intra- and inter-route insertion, intra-route string insertion, 2-opt and swap. Neighborhoods are applied in a cyclic fashion until no improvement is found in one complete cycle. Finally, for each candidate location, the end-points of the routes are used as input for the Weiszfeld algorithm, thus further reducing the total traveled distance. To intensify the search, the best found location becomes the center of the next ellipsoid and the size of the ellipsoid gradually decreases in each iteration. The procedure stops if no improvement can be found in two successive ellipsoids.

The same test data as in (Schwardt and Fischer 2009) is used. The proposed heuristic shows a smaller average gap to the BKS compared to the methods of Schwardt and Dethloff (2005), Schwardt and Fischer (2009), and Salhi and Nagy (2009) and is able to produce nine new best solutions of 15 instances. Moreover, the authors conduct studies that confirm the benefit of all components of their algorithm.

Hashemi Doulabi and Seifi (2013) study a location arc-routing problem (LARP) with uncapacitated facilities on a mixed graph. The objective is to minimize the sum of fixed facility setup and fixed and variable vehicle routing costs. There is an upper bound on the number of facilities to be opened and on the number of routes assigned to each opened facility. Based on previous work by Gouveia et al. (2010) for the mixed Capacitated Arc Routing Problem (CARP), the authors present two arc-variable based formulations, one for the case where one facility is to be opened and one for the case where several facilities may be opened. They also develop an aggregated formulation with fewer variables that can be solved faster. This aggregated formulation is not valid for their problem, but it provides a valid lower bound.

To solve the problem heuristically, an iterative procedure combining an arc-routing and a location-allocation heuristic is proposed. As initial solution, routes visiting only one required link are created and assigned to the closest facility. In each iteration, the arc-routing heuristic receives as input a complete route plan that may violate the upper bound on the number of routes per facility and merges routes one by one. Merging two routes works as follows: One route is selected, and the path from the first to the last required link on the route, say, $(l_f^1, \ldots, l_l^1)$, is inserted between two consecutive required links l_1^2, l_2^2 of another route (consecutive means that no other required link appears between l_1^2 and l_2^2 in the route, but there may be nonrequired links in between). The insertion is performed by linking the end vertex of l_1^2 to the start vertex of l_f^1 and the end vertex of l_l^1 to the start vertex of l_2^2 by means of a shortest path. The location-allocation heuristic iteratively opens and closes facilities and re-assigns existing routes to different facilities. Routing and location-allocation are performed alternately until no improvement is found. The new solution is accepted according to an SA criterion. Then, a neighborhood generator splits up routes, thus forming smaller routes that provide opportunities for merging, and the process is repeated until a stop criterion is met.

Computational experiments are performed with two sets of mixed CARP instances introduced by Belenguer et al. (2006). These instance are interpreted as LARPs by allowing that each vertex be a potential facility. The quality of the lower bound provided by the aggregated formulation is demonstrated by the fact that, for 22 instances with 24–50 vertices and 44–138 links, the lower bound is always equal to the optimal solution value. The heuristic solutions, in turn, are on average 10% above the lower bounds for larger instances with up to 401 vertices and 1056 links.

Stenger et al. (2012) present an extension of the standard LRP, where subcontracted facilities are available that serve assigned customers at a route-independent cost given by the subcontractor. As solution method, the authors present a metaheuristic hybrid of SA and VNS. To generate an initial solution, facilities are opened in a random fashion, customers are assigned to the closest facility with free capacity, and routes for the self-owned facilities are determined by a savings algorithm. In the subsequent location phase, SA uses facility add, drop, and swap moves to improve the facility configuration.

The evaluation of a location move, i.e., the reassignment of customers and the generation of new tours (in case of self-owned facilities), is restricted to a so-called Adjustable Area of Influence (AAI) in order to reduce the computational effort (cp. Nagy and Salhi 1996). The AAI is increased in the course of the SA in order to evaluate the moves in later phases of the SA (when basically only improving solutions are accepted) more precisely. After a location move, customers within the AAI are assigned to their closest facility, and tours for self-owned facilities are generated by the savings algorithm followed by an LS step and two VNS iterations. The shaking step of the VNS is defined on a set of CROSS-exchange neighborhoods (Taillard et al. 1997), and the embedded LS uses 2-opt and relocate moves. If the SA accepts a move, an MDVRP involving all facilities within the AAI is addressed by a complete VNS run. The final solution returned by the SA undergoes another VNS improvement step that is not restricted to the AAI. Infeasible solutions concerning facility and vehicle capacities are allowed and are handled by means of a dynamic penalty mechanism.

Tests on the B instances for the standard LRP show that the presented algorithm is capable of providing good solution quality in short time. Based on the B set, the authors generate test instances with subcontracting options and show that their method is able to exploit this option by producing clearly better solutions compared to a situation where no subcontracting is possible. Finally, the authors address a practice-inspired test case, where the planning starts from the currently established facility configuration and the subcontracting option is given.

Ahn et al. (2012) study a Prize-Collecting LRP (PCLRP). Their problem is motivated by a planning task in space exploration and has several other applications in military operations, sports, and logistics. When viewed from a logistics perspective, the novel feature of the problem, in addition to the location-routing and the prize-collecting aspects, is that for each opened facility, one out of several potential modes of transportation must be selected. Each mode incurs certain costs, induces resource constraints on the routes originating at the facility, such as the vehicle capacity or the maximum route length, and determines which customers can be visited from a facility. The objective is to maximize profit, which consists only of selecting an appropriate subset of customers to be visited; costs are not part of the objective but are bounded from above by a budget constraint.

The problem is solved by heuristic branch and price and by a three-stage heuristic. The heuristic branch-and-price identifies negative reduced cost columns via a modification of an algorithm by Butt and Ryan (1999) and performs incomplete branching by diving along a single branch of the tree (no backtracking). The three-stage heuristic first divides the vertices into groups,

each containing one or more facilities, by computing connected components in an auxiliary graph. Second, it determines a set of facilities and transportation modes for every group and solves an MDVRP with Profits (MDVRPP) for each group and its assigned set of facilities and transportation modes. If the number of facilities in a group is small enough, all facility/mode pairs are enumerated. Otherwise, they are selected randomly based on greedy estimates. The MDVRPP is tackled by solving its LP relaxation with column generation and solving an IP over the resulting set of columns. Third, using a standard solver, it solves an IP that assigns one transportation mode to each group so that the total profit for the overall problem is maximized while ensuring that the selected transportation modes satisfy the budget constraint.
Computational experiments are performed with a set of self-generated random instances and a real-world instance from space exploration. The results show that the heuristic branch-and-price performs better than the three-stage method, finding better or equal solutions for 75% of the instances.
Glicksman and Penn (2008) study the Generalized LRP (GLRP) with uncapacitated facilities and vehicles. The authors develop a $(2 - (1/(|V| - 1)))g_{max}$-approximate algorithm, where g_{max} is the cardinality of the largest group. The algorithm first determines the customers to be visited by solving the LP relaxation of a relaxed arc-variable formulation for the GLRP. With these customers, an instance of a prize-collecting Steiner tree problem is constructed and solved using the algorithm of Goemans and Williamson (1995). The resulting tree is then transformed into a GLRP solution. No computational experiments are performed.
Harks et al. (2013) develop approximation algorithms for several types of LRPs with uncapacitated facilities and homogeneous capacitated vehicles. Besides the split delivery LRP (SDLRP), the authors study the PCLRP, the GLRP, and the corresponding variants of these problems with the possibility of load transshipments at customer locations. Approximation algorithms for the SDLRP, the GLRP, and the PCLRP with approximation factors of 4.38, $4.38g_{max}$, and 6 respectively are presented. For the corresponding problem variants with load transfers, approximation factors of 3.5, $3.5g_{max}$, and 6 are obtained. Note that the algorithms for the non-generalized versions provide a constant-factor approximation.
To solve the SDLRP, the authors compute an approximate solution to a special Uncapacitated FLP (UFLP) and a minimum spanning tree on a modified graph. Then, to obtain an SDLRP solution, the spanning tree is decomposed into subtrees with the property that the sum of the demands of the customers in each subtree is between 50% and 100% of the vehicle capacity. These subtrees are turned into routes by duplicating the tree edges, and the routes are assigned to facilities opened in the UFLP solution or the spanning tree.
This algorithm is modified and used to solve the PCLRP and the GLRP. For the PCLRP, the difficulties are that, for the subtree decomposition to work, the solutions to the UFLP and to the spanning tree must serve the same set of customers, and that the sum of the costs of the solutions to these two subproblems must remain a lower bound for the original problem. The authors resolve these issues by using an approximation algorithm for the prize-collecting UFLP and an LP-based approximation algorithm for the prize-collecting Steiner tree problem to obtain two sets of visited customers. The customers in the intersection of both sets are selected to be visited in the PCLRP solution. To solve the GLRP, the authors describe how a solution of the GLRP can be interpreted as a two-commodity flow on a directed graph. They construct an LP representing such a flow and show how a solution to this LP can be used to select a customer from each group. To handle the possibility of transshipments, the authors modify the subtree decomposition procedure. Any solution returned by one of the algorithms has the property that each customer is visited by vehicles from only one facility. If all customer demands are less than or equal to the vehicle capacity, each customer is visited by exactly one vehicle exactly once. As this latter assumption is usual in the LRP literature, the presented algorithms are suitable for the unsplit delivery, single assignment LRPs commonly studied in the literature.
Computational experiments are performed with the Perl, TB, B, and HKM benchmark instances. The latter are introduced in this paper and contain up to 1,000 facilities and 10,000 customers. For the experiments, each single tour of a solution returned by the approximation algorithm

is improved by the Lin-Kernighan-Helsgaun heuristic. For those Perl, B, and TB instances for which optimal solutions are known from the literature, the average gap is 10%. For the HKM instances, the average gap to lower bounds obtained from the solution of the spanning tree and facility location subproblems is about 60%, a value far better than the theoretical quality guarantee. For the TB and B instances, the run-times are at most 0.02 seconds, and for the HKM instances with 1,000 facilities and 10,000 customers, they range between 5 and 23 minutes.
Schittekat and Sörensen (2009) present a case study in the context of spare parts delivery for a major automobile manufacturer, for which a decision support tool based on the solution of an LRP has been developed. Third-Party Logistics (3PL) providers make bids on delivery regions of the considered company, whose task is to select the set of 3PL providers to operate the intermediate hubs in its distribution network. Thus, the company does not directly have to solve an LRP as the 3PL partners are responsible for the last-mile delivery by vehicle routes. However, the information gained from investigating the LRP allow the company to better estimate the distribution costs of potential 3PL providers, thus enhancing the company's negotiation power in the provider selection process.
The addressed problem considers capacitated facilities and a rich multi-depot VRP featuring heterogeneous vehicles and several constraints such as vehicle capacity, vehicle-dependent site accessibility constraints, time windows, and maximum driving time. The authors propose a TS based on the one described in (Nagy and Salhi 1996) to solve the problem of finding a high-quality facility configuration. The routing solutions are determined by integrating a commercial vehicle routing solver into the decision support tool. A solution (a facility configuration) is encoded as sequence of zeros (facility is closed) and ones (facility is open). The starting solution corresponds to the current configuration used by the company. The TS uses the neighborhoods facility add, drop, and swap. Due to the long run-times of the routing solver, the concept of area of influence proposed by Nagy and Salhi (1996) is used (see Stenger et al. 2012, above). To diversify the search, a frequency memory stores how often each facility is open in previous iterations, and after a number of unsuccessful iterations, the most frequently used facility is deselected, the least used facility is selected, and the inverse moves are included in the tabu list. The key feature of the tool is the generation of a set of diverse, high-quality solutions instead of only one single best solution. This is achieved by keeping a set of elite solutions during the course of the algorithm. The quality of the solutions in the set is guaranteed by only adding solutions whose cost is within a certain percentage of the cost of the best solution found so far. The diversity of the set is ensured by fixing a minimal Hamming distance of a new candidate configuration to any solution in the set. The authors state that this feature is especially useful to analyze alternatives, which increases the negotiation power of the company. Experiments on company data show that the tool is able to provide a set of high-quality solutions with lower cost than the current setting, assessed based on the LRP objective and the routing cost evaluation of the solver. Run-times of the tool are considerable but are deemed adequate by the company due to the strategic importance of the problem.

12 Summary

This section sums up the key insights we gained during our study, concerning problem aspects, practical applications, and algorithmic issues.
Problem aspects. Judging from the number of published papers, the focus of research still lies on the standard LRP, i.e., a deterministic, static, discrete, single-echelon, single-objective problem where each customer must be visited exactly once. Multi-echelon problems form the second most studied variant. In general, there is a trend towards considering more complex and integrated problems: Besides multi-echelon LRPs, multi-objective LRPs and problems incorporating inventory decisions are receiving increased interest from the research community.
Practical applications. Table 2 lists the most important LRP applications and case studies published in the last few years. The table demonstrates the wide applicability of location-routing

models in practice. Moreover, it shows that the facilities to locate in an abstract LRP can be rather diverse objects.

Paper	Problem type	Application
Chan and Baker (2005)	Standard LRP	Depot location and vehicle routing for delivery of classified documents for armed forces in the USA.
Lin and Kwok (2006)	Multi-objective LRP	Vehicle depot location and routing of delivery vehicles for document dispatching in Hong Kong.
Burks (2006)	Two-echelon LRP	Theater distribution problem for military operations: Location of supply bases and vehicle depots, routing of supply vehicles.
Lopes et al. (2008)	Standard LRP	Description of a professional decision support tool for LRP applications.
Caballero et al. (2007)	Multi-objective LRP	Installation of waste incineration facilities considering non-monetary factors such as social rejection by towns close to open facilities, routing of waste collection vehicles.
Marinakis and Marinaki (2008b)	Standard LRP	Location of local delivery facilities for wood distribution in Greece, routing of delivery vehicles.
Ambrosino et al. (2009)	Special LRP variant	Location of local delivery facilities for food distribution for a supermarket chain in Italy, routing of delivery vehicles.
Schittekat and Sörensen (2009)	Standard LRP	Location of hubs and routing of small delivery vehicles in automobile spare parts distribution.
Çetiner et al. (2010)	Many-to-many LRP	Hub location and routing in a real-world mail delivery network.
Stenger et al. (2012)	Standard LRP	Location of parcel delivery self-owned and subcontracted depots, routing of own fleet.
Ahn et al. (2012)	Prize-collecting LRP	Mission planning in space exploration: Plan which missions (routes) to perform with which exploration vehicles from which landing points (potential bases/facilities) on a planet to maximize the scientific value of the information gathered while maintaining a budget constraint.

Table 2: Applications of LRPs and case studies

Algorithmic issues. The crux in solving LRPs, in heuristic as well as exact algorithms, is how to handle the subproblems of location, allocation, and routing. Many different approaches are possible, some of which appear to be particularly attractive, as they are successfully used by several authors. These recurrent solution methods are:

- Exact methods make use of the fact that an optimal solution to the LRP can be computed by minimizing, over all subsets of the set of potential facilities, the opening costs of the facilities in a subset and the costs of an optimal solution to a multi-depot VRP where the depots correspond to the facilities in the subset and have the respective capacities.
- Heuristic approaches often decompose the problem into a location-allocation stage, where the facilities to be opened and assignments of customers to facilities are determined, and a routing stage, where a VRP is solved for each opened facility. Sometimes, allocation decisions are also allowed during the routing stage. In many cases, the two stages are solved iteratively in a feedback loop. It should be noted that both single-solution as well as population-based metaheuristics have been successfully applied to LRPs. For many LRP variants, it has been found that the quality of a solution strongly depends on the opened facilities (see, e.g. Prins et al. 2006a). Therefore, the most successful heuristics intensively search the space of potential facility configurations, often using diversification and intensification phases. GRASP is a very frequently used approach in this context. Determining high-quality routing solutions is not only important for the final solution quality but also for an accurate evaluation of the quality of facility configurations (see, e.g., Stenger et al. 2011). In principle, any fast (MD)VRP heuristic with good solution quality is adequate here.
- Matheuristics often use Lagrangian relaxation.
- Approximation algorithms exploit the relationship between (minimum) spanning or Steiner trees and closed tours (cycles) in graphs to estimate route lengths.
- Multi-echelon LRPs are mostly decomposed by echelon and solved heuristically.

Researchers who intend to work on variants of LRPs might consider the above approaches first.

13 Suggestions for further research

Before presenting our research suggestions, we briefly summarize to what extent the scientific community has taken on the topics proposed by Nagy and Salhi in their 2007 LRP survey. These authors suggest nine topics for future research:

Use of route length formulae instead of vehicle routing algorithms to speed up the routing part of a heuristic: The approximation algorithms described in the papers of Glicksman and Penn (2008), Chen and Chen (2009) and Harks et al. (2013) rely on this principle. Apart from that, only Albareda-Sambola et al. (2012) exploit the idea.

Dynamic and stochastic problems. We found four papers on stochastic and four on fuzzy LRPs (see Section 8). Compared to the number of papers on deterministic problems, this is clearly expandable, all the more so as only a small share of real-world applications is of deterministic nature. Six papers in this review consider periodic and multi-period problems (see Section 6).

Planar location. Literature on problems where the potential facilities may be located anywhere in the plane is still scarce: Only two new papers (Schwardt and Fischer 2009, Manzour-al-Ajdad et al. 2012) treat such problems since the last review (see Section 11). From our point of view, the variant with multiple facilities to locate, or extensions such as considering forbidden regions deserve the attention of the research community.

Integrated methods in logistics. Nagy and Salhi state that it would be interesting to combine location-routing with other aspects of logistics, and mention inventory and packing aspects in particular. As Section 10 shows, the former type of problems has indeed been studied more often in recent years, but we think that the topic still needs further research. We found no papers dealing with LRPs incorporating packing aspects.

Multi-objective LRPs. This problem variant has received quite some attention, see Section 9, but in our opinion is also far from being exhausted.

Competitive LRPs. Nagy and Salhi point out that competitive location theory is a well-established field but that there is no work on competitive LRPs. As far as we know, this is still the case.

Eulerian location/location arc-routing. The sole paper on LARPs we found is (Hashemi Doulabi and Seifi 2013, Section 11).

Hybrid methodologies. Nagy and Salhi point out the fragmentation of LRP research into various strands and request to unite different methodologies. In particular, they advocate the combination of exact and heuristic methods. The widespread use of benchmarking in the recent literature is a major step to better connecting different research strands, and the numerous matheuristic approaches described in this review provide evidence that exact and heuristic methods have started to unite in the field of LRPs.

Modeling complex situations. Nagy and Salhi request that more complex and realistic problems be studied. As can be seen in Section 11, there is actually a trend to considering more comprehensive and integrated models.

This short discussion shows that several of the research gaps listed by Nagy and Salhi have not yet been filled.

In addition to the topics just mentioned, we propose the following potential areas for future research, dealing with methodological as well as modeling aspects. From a methodological point of view, these are:

Systematic techniques for parameter optimization and design of experiments: To find suitable values for the parameters of an algorithm, it is still common to perform what most authors call 'preliminary testing'. Systematic and documented approaches are rare. By performing sophisticated statistical tests and reporting on the results, the papers of Burks (2006) and Nguyen et al. (2010, 2012b,a) constitute the notable exceptions.

Algorithm evaluation criteria: It is surely a valuable and nontrivial task to devise and fine-tune an algorithm so that it performs well (with respect to solution quality and run-time) on a given set of benchmark instances. As mentioned, this is what has increasingly been done in the last few years. The general scientific and practical value of a solution approach, though, is affected by additional criteria, such as simplicity (How easy is it to understand the algorithmic principle?), flexibility (How easy is it to include additional constraints?), and robustness (Does the algorithm compute high-quality solutions for different instances?) (see Cordeau et al. 2002, Bräysy and Gendreau 2005). The only reference we are aware of that conducts a thorough study of these criteria is (Burks 2006), who uses a design of experiments (DOE) approach. For future works, similar analyses are recommended to obtain deeper insights on the usefulness of algorithms under different conditions.

Causal performance analysis of metaheuristic approaches: In general, absolutely no light is shed on the important question why a certain metaheuristic performs better than other approaches with a comparable degree of sophistication. There is not a single paper that elaborates on this issue. This is in marked contrast to discussions on reasons for the strength of MIP formulations and corresponding exact algorithms. Progress in this area would be very valuable.

Algorithms for large-scale problems. As pointed out in Section 4.3, many math- and metaheuristics have been developed that all achieve high-quality results on the medium-sized standard LRP benchmarks TB, B, and PPW. Apart from Harks et al. (2013), who develop approximation algorithms for several types of LRPs and successfully solve large instances, and Alvim and Taillard (2013), who tackle extremely large instances of an LRP model without facility capacity constraints, we are not aware of methods specifically designed for large-scale problems. Such problems, however, better reflect the characteristics of practical applications, and it appears a worthwhile but challenging task to develop heuristic methods able to obtain convincing solution quality within acceptable run-times. To this end, developing benchmark instances with a size lying between the 100–200 customers of most benchmark sets in Table 1 and the 10,000 customers of the HKM instances could be helpful.

From a modeling point of view, we propose the following research topics:

Important but rarely-studied variants. Despite their theoretical and practical relevance, pickup-and-delivery LRPs, generalized LRPs, split delivery LRPs, and LRPs with time windows have been rarely considered until now. With the exemption of generalized problems, this is in contrast to the literature on vehicle routing, where the respective problem variants have received a considerable amount of attention (see, e.g, Parragh et al. 2008a,b, Archetti and Speranza 2008, Bräysy and Gendreau 2005, Baldacci et al. 2012).

Multi-echelon LRPs with space-time synchronization. It is noteworthy that most papers on multi-echelon LRPs ignore the temporal aspect of the load transfers that occur in such situations. This may be justified for strategic or tactical applications. In an operative setting, though, the temporal aspect of transshipments must be taken into account and requires synchronization of operations in space *and* time (Drexl 2012). There is hardly any work on this admittedly very difficult topic.

Integration of location, routing, and revenue management. Although there is a trend to more integrated and complex models, as mentioned above, and although the revenue aspect of long-, medium-, and short-term economic activities is studied by an entire branch of operations research (Phillips 2005), the usual objective in LRPs is still cost minimization. We are unaware of any papers on the integration of revenue management aspects into location-routing models. In our opinion, contributions to this field would be very valuable, as revenue management is meanwhile successfully employed in many other logistics areas (Talluri and Van Ryzin 2008). We are convinced that integrating revenue management into LRP models will provide highly interesting and challenging research questions.

All in all, we hope that the present survey provides useful information for researchers and will motivate further work in the field of location-routing.

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Appendix

Summary of abbreviations

2E-LRP	Two-echelon LRP
2E-UCFLP	Two-echelon UCFLP
3PL	Third-party logistics
AAI	Adjustable area of influence
ALNS	Adaptive large neighborhood search
AMP	Adaptive memory procedure
ATS	Adaptive TS
BKS	Best known solution
CARP	Capacitated arc routing problem
CFLP	Capacitated FLP
DOE	Design of experiments
ELS/ILS	Evolutionary/Iterated LS
ENS	Expanding neighborhood search
ESPPRC	Elementary shortest path problem with resource constraints
ETS	Elite TS
FLP	Facility location problem
GA	Genetic algorithm
GLRP	Generalized LRP
GRASP	Greedy randomized adaptive search procedure
GTS	Granular TS
ILS	Iterated LS
IP	Integer program(ming)
LARP	Location arc-routing problem
LP	Linear program(ming)
LR	Lagrangian relaxation
LRP	Location-routing problem
LRPSPD	LRP with simultaneous pickup and delivery
LS	Local search
M(C)DVRP	Multi-depot VRP (with capacitated depots)
MDVRPP	Multi-depot VRP with profits
MIP	Mixed integer program(ming)
MMLRP	Many-to-many LRP
MOSS	Multi-objective SS
NDP	Network design problem
NE-LRP	N-echelon LRP
PCLRP	Prize-collecting LRP
PLRP	Periodic LRP
PR	Path relinking
RECWA	Randomized extended Clarke-Wright algorithm
SA	Simulated annealing
SCDP	Supply chain design problem
SDLRP	Split delivery LRP
SS	Scatter search
SSCFLP	Single-source CFLP
TS	Tabu search
TSP	Traveling salesman problem
UFLP	Uncapacitated FLP
VLNS	Very large-scale neighborhood search
VND	Variable neighborhood descent
VNS	Variable neighborhood search
VRP	Vehicle routing problem

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