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Other-Regarding Preferences, Concerns for Others' Opportunities and Violations of Stochastic Dominance—A Choice Theoretic Analysis

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Abstract

Decision makers with other-regarding preferences may care not just about others' *ex-post outcomes* but also about their *ex-ante opportunities*. In environments of risk, so doing may lead to violations of the property of *stochastic dominance* that is at the heart of existing theories of decision making under risk, expected utility and non-expected utility alike. We propose choice theoretic foundations for a decision model of other-regarding preferences that accommodates such violations. Our decision model provides a representation of a decision maker's preferences over lotteries on an allocation-space that is based on (*i*) a *baseline expected utility evaluation* and (*ii*) an *adjustment* that corrects the expected utility evaluation to account for the decision maker's concerns about others' overall opportunities under a lottery.

1 Introduction

In recent years the term *other-regarding preferences* has featured prominently in the economics literature. A decision maker's preferences may be termed as other-regarding if she is concerned not just about her own prospects but others' as well. Economists have collected an impressive body of experimental evidence that is strongly indicative of the fact that such concerns matter for many decision makers.[1] At the same time, they have shown that introducing such concerns into economic models produce novel insights that are of qualitative

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[1]For a recent survey on the experimental literature refer to Cooper and Kagel (2012)

and quantitative significance.[2] It will be fair to say that, as it stands, the literature on other-regarding preferences has had a significant influence on recent economic thought.

Although cognizant of this influence, in this paper, our starting point is a critical evaluation of this literature from a choice theoretic perspective. It will be our contention that adequate focus has not been placed on understanding the choice theoretic properties of other-regarding preferences. Indeed, we shall argue that choice behavior of decision makers with such preferences may be at serious odds with some of the central tenets of models of decision making that economists typically rely on. What's more, many of the popular approaches to modeling other-regarding preferences that have been proposed in the literature—for example, Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Charness and Rabin (2002), amongst others—have not addressed this challenge that such preferences pose at a choice theoretic level. It is with the goal of addressing this gap in the literature that, in this paper, we provide choice theoretic foundations for a new model of other-regarding preferences.

What is the challenge that other-regarding preferences pose for existing models of decision making? To answer this question, consider a decision maker, Beth, who has to decide between two divisions of \$100 between herself and another person: under the first allocation—let us call it x—she gets \$90 and the other person gets \$10, whereas, under the second—let us call it y—she gets the the entire money. It is conceivable that she may choose the first allocation, x, as she may find the second blatantly unfair towards the other person. Next, suppose that Beth has to choose between two lotteries, p and q. Under both lotteries the other person gets the entire money with 95% chance, whereas, with 5% chance, the allocation is x under p, and y under q. Which of the two lotteries will she choose? We contend that it may not be unreasonable to imagine that her choices may now show a "reversal" and she may indeed choose q over p. Her reasoning may go as follows: "In the first choice, I feel good about sacrificing \$10 as it results in a fairer deal for the other person. However, when I consider the second choice, I am not inclined towards making a similar sacrifice in the .5-probability event, which choosing p over q entails, as the overall prospects are overwhelmingly in the other persons' favor to begin with. After all, it is almost certain that he will end up with the entire money, and so, if against the odds, we were to end up in the .5-probability event, I would not consider it unfair towards him if he receives nothing—*he has had his chance!*"

What the example illustrates is that for decision makers like Beth, who show concern about others, there may be two distinct considerations when it comes to expressing this concern. The first is about others' *ex-post outcomes* while the second is about their overall *ex-ante opportunities* (or chances), and the choices of such decision makers, it appears, is

[2]For instance, economists have appealed to other-regarding preferences to deepen their understanding of such matters as Ricardian equivalence (Andreoni, 1989), the equity premium puzzle (Abel, 1990), the difference in redistribution policy between United States and Western Europe (Alesina and Angeletos, 2005), amongst others.

determined by the interaction between the two. For instance, as in the example, decision makers who may otherwise be resistant to unfair outcomes for others, may be less opposed to them if they deem that the others in question had their fair shot at garnering a good outcome. Indeed, this idea that individuals deserve a fair shot and, therefore, in evaluating risky social prospects, the ex-ante opportunities available to them should matter, separate from what ex-post outcomes may be, is fairly well documented. At a societal level, this idea has often been used as a guiding principle for public policy and has, at times, even been codified in the law.[3] So, it should not surprise us if individual decision makers appeal to this very idea, as well, when it comes to evaluating risky social prospects. At any rate, this insight has been validated at the level of individual decision making in recent experimental work—amongst others, in Bolton, Brandts and Ockenfels (2005), Kircher, Ludwig and Sandroni (2010) and Krawczyk and Le Lec (2010).

Given that social situations of risk allow a decision maker to distinguish between the ex-post outcomes of others and their ex-ante opportunities or chances, it naturally begs the question: Are existing theories of decision making under risk that economists typically rely on adequate to accommodate the choice behavior of decision makers for whom both the ex-ante and ex-post considerations matter? As it turns out, from the perspective of these theories the choices of such decision makers may appear puzzling. To understand why this is the case, consider Beth's example. Suppose she were in possession of the lottery q. Now consider replacing this lottery with the lottery p. What does this replacement entail? Observe that what we have done is move a 5% chance from her less preferred outcome—the allocation in which she gets the entire money—to her more preferred outcome—the allocation in which she gets 90 and the other person gets 10—whereas with 95% chance the other person gets the entire money under both p and q and hence the outcome remains the same. Standard theories of decision making under risk, whether it be expected utility or models of non-expected utility, have an unambiguous conclusion to make in this case—Beth should be made strictly better off as a result of this replacement. This is the conclusion warranted by the property of *stochastic dominance*, which is at the heart of these theories of decision making under risk.[4] It is worth noting that stochastic dominance is a more fundamental

[3]It is not uncommon, for instance, to find lotteries being used as a way of allocating scare resources like public housing, medical resources (eg. kidney transplants), admission to educational institutions, athletic drafts (eg. the National Basketball Association), (avoiding) military drafts, US green cards etc. On the point about the law, refer to Bolton, Brandts and Ockenfels (2005), who provide an illustration of this idea being put to practise in the US v. Holmes case.

[4]Formally, suppose X is a set of outcomes, $\Delta(X)$ is the set of lotteries on X, and the decision maker has a preference relation $\succcurlyeq$ over $\Delta(X)$. Then we will say that a lottery p in $\Delta(X)$ first order stochastically dominates another lottery p' with respect to $\succcurlyeq$ if for all $x \in X$, the probability that p assigns to outcomes that are at least as good as x is no less than the corresponding probability under p', and for some x it is strictly greater. *Stochastic dominance* says that if p first order stochastically dominates p', then p is strictly preferred to p'.

assumption than the famous *independence* condition of expected utility theory. Violation of stochastic dominance implies a violation of independence, but not vice versa.[5] Non-expected utility models (eg., rank dependent utility, betweenness based theories, generalized expected utility, generalized prospect theory) generalize expected utility by giving up the independence condition, but retaining stochastic dominance. Viewed abstractly, the property of stochastic dominance, which is a natural generalization of the idea that "more (of a good) is better" to environments of risk, seems almost unchallengeable and synonymous with rationality—after all, if in every event our decision maker is getting better off, how could it not be that she is better off overall? Hence, from the perspective of standard theories of decision making Beth's choices appear puzzling for she would much rather have the lottery q than p, thus violating stochastic dominance. Accordingly, neither expected utility nor models of non-expected utility may be able to accommodate her choices, as well as that of those like her, who, in evaluating the risk faced by others, may care not just about their ex-post outcomes, but also about their ex-ante opportunities.

That is the challenge that other-regarding preferences pose at a foundational level. Lines of reasoning, like stochastic dominance, which seem natural and compelling in the context of "private" decision problems do not seem to carry over well into decision problems in which social concerns matter. In other words, the way individuals evaluate "social goods" may be different from the way they evaluate "private goods." In particular, observe that there is a key behavioral assumption underlying the property of stochastic dominance—it requires a decision maker's behavior to conform to a basic form of consequentialism, namely, that her ranking of outcomes should be independent of the stochastic process that generates these outcomes. On the other hand, decision makers with other-regarding concerns may be fundamentally non-consequentialists, and their ranking of ex-post outcomes may intrinsically depend on the stochastic process that generates these outcomes, as that influences their perceptions of the overall opportunities available to the others.

The above discussion, in turn, casts serious doubts on the strategy that has been pursued in many of the popular approaches to modeling other-regarding preferences like Fehr and Schmitt (1999), Bolton and Ockenfels (2000), Charness and Rabin (2002), amongst others. In these approaches, the goal has been to specify functional forms for "social utility functions," defined over sure allocations, that seek to capture social motivations like altruism, inequity

[5]For instance, Allais' famous example violates independence but not stochastic dominance.

aversion, inefficiency aversion etc.[6] When it comes to the issue of specifying preferences in environments featuring risk, it appears that the implicit assumption all along has been that these models can be extended to such environments by taking a ready-made model of decision making under risk like expected utility. What has not been recognized in these approaches is the possibility discussed above that none of these ready-made models may be appropriate for decision makers with other-regarding preferences. The lesson, then, that we take away from this discussion is that ad-hoc ways of modeling decision makers with other-regarding preferences when faced with risk are unlikely to be too profitable. As a research project, the goal should be to think systematically about the choice-theoretic properties that such decision makers may satisfy, and *derive* preference representations that follow from these properties. That is the objective that we pursue in this paper.

We consider a decision maker's primitive preferences over lotteries on a well-defined space of allocations[7] ("allocation-lotteries", for short). Our goal is to propose a parsimonious decision model that accommodates the choice behavior of decision makers like Beth who, besides caring about their own end-outcomes, care both about others' end-outcomes as well as their ex-ante opportunities. Here, we interpret parsimonious to mean a decision model that is conceptually a minor deviation from the expected utility model, and reduces to it in the special case of decision makers who do not express a separate concern for others' ex-ante opportunities. More specifically, what we propose is a decision model that explains the choices of a decision maker over allocation-lotteries by precisely identifying the *adjustment* that she makes to a *baseline expected utility* evaluation of lotteries whenever that evaluation, in her assessment, does not adequately account for the overall opportunities available to the others.[8] What we formally establish in this paper are the precise conditions, or axioms, on the decision maker's choice behavior that allow us to derive such a model from (in-principle) observable choices, including, identifying the expected utility functional and the adjustment essentially uniquely.

To put things more formally, suppose there is a decision maker (DM) and n other individuals about whose prospects she may potentially care. Let Z denote the set of outcomes for DM, A the set of outcome-vectors for the others (others' outcomes, for short), so that $Z \times A$

[6]For instance, Fehr and Schmidt propose the following social utility function to evaluate the utility derived by individual 1 in a two individual world. Suppose individual 1 receives the outcome x_1 and individual 2 receives the outcome x_2 (x_1, $x_2 \in \mathbb{R}_+$). Then individual 1's utility is given by:

$$U_1(x_1, x_2) = x_1 \text{ - } \mu.\max\{x_2 - x_1,\, 0\} \text{ - } \mu'.\max\{x_1 - x_2,\, 0\},\ \mu,\ \mu' > 0.$$

The basic idea behind this functional form is to incorporate a notion of inequity aversion. The decision maker receives 'utility' from her own outcome x_1, but receives 'dis-utility' from inequities in the final allocation.

[7]We use the term allocation in its standard usage—it denotes a list of outcomes, one for each individual in society.

[8]Although the contexts are different, our use of the term "adjustment" is inspired by Siniscalchi (2009).

denotes the set of allocations. DM has preferences over the set $\Delta(Z\times A)$ of (simple) lotteries on $Z\times A$, which we refer to as allocation-lotteries. Under our proposed representation of DM's preferences, which we refer to as the *adjusted expected utility (AEU)* representation, we identify two real valued functions w: $Z\times A \to \mathbb{R}$ and g :$\Delta(A) \to \mathbb{R}$, where $\Delta(A)$ is the set of simple lotteries over A, such that DM's preferences can be represented by the function W: $\Delta(Z \times A) \to \mathbb{R}$ given by

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A),$$

where for any allocation-lottery $p \in \Delta(Z \times A)$, $p(z,a)$ specifies the probability that p assigns to the allocation (z,a) and $p_A \in \Delta(A)$ is the the marginal measure of p over A.

Under the representation, the function w represents DM's ranking of ex-post allocations. We may, accordingly, interpret the first term, $\sum_{(z,a)\in Z\times A} p(z,a)w(z,a)$, as DM's *baseline expected utility* evaluation of the lottery p. This function w is unique up to positive affine transformation. What about the second term? The function g captures what we call the *adjustment*. For any allocation-lottery, the adjustment associated with that lottery is a function of the marginal measure over the others' outcomes under that lottery. This marginal measure serves as a "proxy" for the ex-ante opportunities available to the the others under that lottery. The role of the adjustment is to quantify by how much the baseline expected utility evaluation of a lottery needs to be adjusted or corrected to better reflect the decision maker's assessment of the overall opportunities available to the others under that lottery. As an example, consider Beth's evaluation of the allocation-lottery q, which results in the allocation $(0, 100)$ with probability .95 and the allocation $(100, 0)$ with probability .05. In this case, by her expected utility evaluation, Beth would evaluate the two allocations $(0, 100)$ and $(100, 0)$ using the function w and then take the probability weighted average of these payoffs. However, she may consider this expected utility evaluation to be understating her subjective valuation of this lottery. For instance, she may consider that $w(100, 0)$, which specifies her payoff when the allocation $(100, 0)$ realizes for sure, is not an accurate reflection of her assessment of the situation when this allocation realizes with a low 5% chance under the lottery q. In part, the payoff $w(100, 0)$ may be "lowered" by her concern for the fact that the other person gets nothing (for sure) under this allocation. However, when this allocation realizes with only a 5% chance under the lottery q, where with overwhelming odds the other person can expect to receive a far better outcome, her concerns for the other person receiving nothing are nowhere as pronounced. Accordingly, based on her consideration of the overall opportunities available to the other person, Beth may attach a positive adjustment to the lottery q, which would imply that her payoff from the lottery exceeds her expected utility evaluation of the lottery.[9] Under the representation, as the above reasoning suggests, the

[9] To completely rationalize Beth's choices through our representation, note that her choice of the allocation

adjustment associated with an allocation-lottery in which the others gets some outcome-vector for sure is zero as in this case the actual outcome that the others get is the same as the overall opportunities that they have. The adjustment associated with allocation-lotteries in which the others face non-degenerate risk can be either positive, negative or zero and the representation uniquely pins down the sign of the adjustment associated with any such lottery.

After providing the axioms for an AEU representation, we explore the question of the degree to which the decision maker's "immediate" concerns for her own outcomes can be separated from her social concerns in the context of such an AEU representation. Following Arrow, we call the first concern her *tastes* and the second her *standards of equity.* We first consider the case when such a separation can be made in the strong sense of the decision maker having independent risk preferences over lotteries on her own outcomes ("own-lotteries") as well as lotteries over others' outcomes ("others'-lotteries"). In this context, we identify an additional axiom, which when added to the list of axioms needed for an AEU representation, provides us with a special case of an AEU representation under which the decision maker has independent risk preferences over her own-lotteries as well as others'-lotteries. We refer to such a representation as an *independent risk preference AEU representation.* The high degree of separability between the decision makers' evaluation of her own outcomes from her social concerns that this representation provides, however, comes at a cost. Such a representation is unable to accommodate reasonable notions of "ex-post fairness" as Fudenberg and Levine (2011) have pointed out. To address this concern we develop a representation, which we refer to as *taste-equity separable AEU representation*, that weakens the degree of this separability. Both these special cases of AEU representations identify a function $v : A \to \mathbb{R}$ over the others' outcomes that represents the decision maker's ranking of these outcomes. Accordingly, these representations provide a basis for the implicit assumption we made above while motivating the adjustment that the decision maker has a consistent preference ranking over others' outcomes.

We use this function v that represents a decision maker's ranking of others' outcomes to propose a conjecture for how she may make adjustments. Under this specification of the adjustment the decision maker determines the adjustment associated with an allocation-

$(90, 10)$ over $(100, 0)$ implies that $w(90, 10) > w(100, 0)$. At the same time her choice of the lottery q over the lottery p implies that:

$$.95w(0, 100) + .05w(100, 0) + g([100, .95;\ 0, .05]) > .95w(0, 100) + .05w(90, 10) + g([100, .95;\ 10, .05]),$$

where $[100, .95;\ 0, .05]$ and $[100, .95;\ 10, .05]$ respectively denote the marginal measure of q and p over the other person's outcomes. This in turn implies that to rationalize Beth's choice of q over p it must be that

$$g([100, .95;\ 0, .05]) - g([100, .95;\ 10, .05]) > .05(w(90, 10) - w(100, 0)).$$

lottery p, which generates a marginal lottery $p_A = [a^1,\ \lambda^1;\ .\ .\ .\ ;\ a^N,\ \lambda^N]$ over others' outcomes in the following manner:

$$g(p_A) = \sum_{n=1}^{N} \lambda_n.\max\{\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}}) - v(a^n), 0\}.$$

Under this specification of the adjustment the decision maker considers the (subjective) average payoff under p_A, given by $\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}})$, to be indicative of the overall opportunities available to the others under the lottery p. Whenever, a realized outcome for the others, say $a^n \in A$, is such that $v(a^n)$ is less than this average payoff, she makes an upward adjustment that is equal to the difference between $\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}})$ and $v(a^n)$. On the other hand, when the realized outcome for the others, a^n, is such that $v(a^n)$ is no less than this average payoff, she makes no adjustments. The total adjustment is then determined by the probability weighted average of these individual adjustments. Because of the asymmetry that the decision maker shows in terms of her evaluation of events in which others get a below-average outcome as compared to events in which they receive an above-average outcome, we refer to this particular type of adjustment as *asymmetric adjustment* and we propose a choice-theoretic characterization of such an adjustment.

The paper is organized as follows. Section 2 sets up the framework. Section 3 axiomatizes the AEU representation. Section 4 identifies the additional axioms required to establish the two classes of AEU representations, namely, independent risk preference and tastes-equity separable. Section 5 provides the choice theoretic characterization of asymmetric adjustments. Finally, section 6 concludes by emphasizing the need to engage the literature on other-regarding preferences with that on nonseparable preferences in environments of risk. All proofs are provided in the Appendix.

2 The Framework

2.1 Preliminaries

We assume that our stylized society comprises of a decision maker (DM) and n other individuals, denoted $1, \ .\ .\ .\ , n$, about whose prospects DM may care. Associated with each individual is a well defined set of outcomes. We denote the set of DM's outcomes by Z and those of individual i by A_i, $i = 1, \ .\ .\ .\ , n$. Further, we let $A = \prod_{i=1}^{n} A_i$ denote the set of outcome-vectors for the others (others' outcomes, for short). Accordingly, $Z \times A$ will denote the set of allocations for this society. We will denote generic elements of Z by z, z' etc., that of A by a, a' etc. and that of $Z \times A$ by (z, a), (z', a') etc. We denote the set of simple probability measures (simple lotteries, or just lotteries, for short) on the sets $Z \times A$, Z and A by Δ, $\Delta(Z)$ and $\Delta(A)$ respectively. We will refer to elements of Δ as allocation-lotteries and denote generic elements of this set by p, q etc. For any allocation-lottery $p \in \Delta$, we will denote the marginal probability measure of p over A by $p_A \in \Delta(A)$. As mentioned in the introduction, we take the marginal measure p_A to be indicative of the ex-ante opportunities or chance available to the others under p. Since any lottery in $\Delta(A)$ is the marginal measure over A of some allocation-lottery in Δ, to economize on notation, we will denote generic elements of $\Delta(A)$ by p_A, q_A etc. For any $p \in \Delta$, $p(z, a)$ shall denote the probability that p assigns to the outcome $(z, a) \in Z \times A$. Similarly $p_A(a)$ shall denote the probability that p_A assigns to $a \in A$. We define a convex combination of lotteries in the standard way.[10] We will denote degenerate lotteries by putting the outcome that the lottery puts unit probability on within a [.]-bracket. For instance, $[(z, a)] \in \Delta$ and $[a] \in \Delta(A)$ denotes degenerate lotteries that assigns probability 1 to the outcomes (z, a) and a respectively.

2.2 Preference

DM's preferences are given by a weak order (a binary relation that is complete and transitive) $\succcurlyeq$ on the set Δ. The symmetric and asymmetric components of $\succcurlyeq$ are defined in the usual way and denoted by $\sim$ and $\succ$ respectively.

3 The Adjusted Expected Utility Model

3.1 Axioms

We now provide a set of axioms for the adjusted expected utility (AEU) model. To begin with, we require that DM's preferences are complete and transitive.

AXIOM 1. Weak Order
$\succcurlyeq$ *is complete and transitive.*

Our next axiom puts restrictions on how good or bad lotteries can be in DM's subjective evaluation.

AXIOM 2. Archimedean
For any p, q, $r \in \Delta$, such that $p \succ q \succ r$ and $p_A = r_A$, there exists $\overline{\lambda}$, $\underline{\lambda} \in (0, 1)$, such that

$$\overline{\lambda}p + (1 - \overline{\lambda})r \succ q \succ \underline{\lambda}p + (1 - \underline{\lambda})r.$$

This is the standard Archimedean condition of the expected utility setting (see Kreps 1988) with the qualification, though, that it is required to hold only when the "extreme"

[10] For instance, if $p^1, \ldots, p^K \in \Delta$, and $\lambda^1, \ldots, \lambda^K$ are constants in $[0, 1]$ that sum to 1, then $\sum_{k=1}^{K} \lambda^k p^k$ denotes an element in Δ that gives the outcome $(z, a) \in Z \times A$ with probability $\sum_{k=1}^{K} \lambda^k p^k(z, a)$.

lotteries p and r provide the same ex-ante opportunities to the others, i.e., $p_A = r_A$. With this qualification in place, the interpretation is the standard one: DM cannot consider p to be so good that any compound lottery $\underline{\lambda}p + (1-\underline{\lambda})r$ involving p and r is always considered better than q, no matter how small a probability $\underline{\lambda}$ is put on p. Similarly, DM cannot consider r to be so bad that that any compound lottery $\overline{\lambda}p + (1-\overline{\lambda})r$ involving p and r is always considered worse than q, no matter how large a probability $\overline{\lambda}$ is put on p. Why is the qualification that p and r provide the same ex-ante opportunities to the others needed? Observe that in our setting, when we take the probability mixture of two lotteries to form a compound lottery, the ex-ante opportunities available to the others under the compound lottery typically differs from that under the component sub-lotteries. So, preferences involving the compound lottery can no longer be used to make inferences about these sub-lotteries, which is precisely what we did in the interpretation of the axiom. However, if we take the probability mixture of two lotteries like p and r that provide the others with the same ex-ante opportunities, then the ex-ante opportunities available to them under any compound lottery $\lambda p + (1-\lambda)r$ is also the same.[11] In this case, preferences involving $\lambda p + (1-\lambda)r$ indeed do reflect DM's assessment of p and r as individual lotteries and, accordingly, the interpretation of the axiom that we provided above is consistent.

Our next axiom, called *opportunity-comparable independence*, adapts the independence condition of the expected utility model to our current setting. As mentioned earlier, in our setting, when we take the probability mixture of two lotteries to form a compound lottery, the ex-ante opportunities available to the others under the compound lottery typically differs from that under the component sub-lotteries. For this reason, the independence condition, which requires separability of preferences across disjoint events, typically fails to hold in our setting as DM's ranking of any two lotteries need not carry over to situations when these lotteries are sub-lotteries of other lotteries, given that others' opportunities may vary across these comparisons. The axiom that we now provide identifies the situations under which the separability implied in the independence condition continues to hold in our set-up.

AXIOM 3. Opportunity-Comparable Independence
If p', $q' \in \Delta$ are such that $p' \sim q'$, then for any $\lambda \in [0,1]$,

$$p \succcurlyeq q \text{ if and only if } \lambda p + (1-\lambda)p' \succcurlyeq \lambda q + (1-\lambda)q',$$

whenever p, $q \in \Delta$ are such that either (i) $p_A = p'_A$, $q_A = q'_A$ or (ii) $p_A = q_A$, $p'_A = q'_A$.

Consider the two cases (*i*) $p_A = p'_A$, $q_A = q'_A$ and (*ii*) $p_A = q_A$, $p'_A = q'_A$. Under the first, the difference in others' opportunities between the lotteries $\lambda p + (1-\lambda)p'$ and $\lambda q + (1-\lambda)q'$ is the same as that between p and q on one hand and p' and q' on the other.[12] Under the

[11]since $p_A = r_A$, it follows that $(\lambda p + (1-\lambda)r)_A = p_A = r_A$
[12]In this case, $(\lambda p + (1-\lambda)p')_A = p_A = p'_A$ and $(\lambda q + (1-\lambda)q')_A = q_A = q'_A$.

second, $\lambda p + (1-\lambda)p'$ and $\lambda q + (1-\lambda)q'$ provide the same ex-ante opportunities to the others as do p and q on one hand, and p' and q' on the other.[13] Accordingly, under both these cases, in comparing the lotteries $\lambda p + (1-\lambda)p'$ and $\lambda q + (1-\lambda)q'$, DM can evaluate the sub-lotteries in the λ-probability event, namely, p and q, and those in the $(1-\lambda)$-probability event, namely, p' and q', just as she would if she were evaluating these by themselves. Since p' and q' are indifferent, she should, therefore, prefer $\lambda p + (1-\lambda)p'$ to $\lambda q + (1-\lambda)q'$ if and only if she prefers p to q.

Our next axiom, called *consistent evaluation of opportunities*, builds on the logic of separability developed in the last axiom and introduces a consistency requirement on how concerns for others' opportunities influence DM's evaluation of allocation-lotteries. To understand the content of this axiom, consider allocation-lotteries $\widetilde{p}$, $\widetilde{p}'$, $\widetilde{q}$, $\widetilde{q}' \in \Delta$ for which $\widetilde{p}_A = \widetilde{p}'_A$, $\widetilde{q}_A = \widetilde{q}'_A$ and $\widetilde{p} \sim \widetilde{q}$, $\widetilde{p}' \sim \widetilde{q}'$. Based on these preferences, we may infer that DM considers the (preference) difference between $\widetilde{p}$ and $\widetilde{p}'$—i.e., her subjective "gain" or "loss" from replacing $\widetilde{p}'$ with $\widetilde{p}$—to be the same as that between $\widetilde{q}$ and $\widetilde{q}'$. Next, observe that, since others' ex-ante opportunities are the same under $\widetilde{p}$ and $\widetilde{p}'$ on the one hand and $\widetilde{q}$ and $\widetilde{q}'$ on the other, the same is true for the allocation-lotteries $.5\widetilde{p} + .5\widetilde{q}'$ and $.5\widetilde{p}' + .5\widetilde{q}$. Accordingly, in comparing $.5\widetilde{p} + .5\widetilde{q}'$ and $.5\widetilde{p}' + .5\widetilde{q}$, DM may think of the change involved in replacing the latter with the former as one where, with even chances, either $\widetilde{p}'$ is replaced by $\widetilde{p}$ or $\widetilde{q}$ is replaced by $\widetilde{q}'$ and her evaluation of the relative merits of $\widetilde{p}$ versus $\widetilde{p}'$ on the one hand and $\widetilde{q}$ versus $\widetilde{q}'$ on the other, in this context, is the same as when these comparisons are made individually by themselves. Hence, it stands to reason that DM's preference over $.5\widetilde{p} + .5\widetilde{q}'$ and $.5\widetilde{p}' + .5\widetilde{q}$ is based on her assessment of the difference between $\widetilde{p}$ and $\widetilde{p}'$ to that between $\widetilde{q}$ and $\widetilde{q}'$ and she prefers $.5\widetilde{p} + .5\widetilde{q}'$ to $.5\widetilde{p}' + .5\widetilde{q}$ if and only if she considers the difference between $\widetilde{p}$ and $\widetilde{p}'$ to exceed that between $\widetilde{q}$ and $\widetilde{q}'$. But we had inferred earlier, based on the preferences $\widetilde{p} \sim \widetilde{q}$, $\widetilde{p}' \sim \widetilde{q}'$, that DM considers the difference between $\widetilde{p}$ and $\widetilde{p}'$ to be the same as that between $\widetilde{q}$ and $\widetilde{q}'$. Accordingly, DM should be indifferent between $.5\widetilde{p} + .5\widetilde{q}'$ and $.5\widetilde{p}' + .5\widetilde{q}$. The following axiom strengthens the scope of such reasoning.

AXIOM 4. Consistent Evaluation of Opportunities
Let $\widetilde{p}$, $\widetilde{p}'$, $\widetilde{q}$, $\widetilde{q}' \in \Delta$ *be such that* $\widetilde{p}_A=\widetilde{p}'_A$, $\widetilde{q}_A=\widetilde{q}'_A$ *and* $\widetilde{p} \sim \widetilde{q}$, $\widetilde{p}' \sim \widetilde{q}'$. *If* q, $q' \in \Delta$ *with* $q_A=q'_A=\widetilde{q}_A$ *are such that* $.5q+.5\widetilde{q}' \sim .5q'+.5\widetilde{q}$, *then then for any* p, $p' \in \Delta$ *with* $p_A=p'_A=\widetilde{p}_A$,

$$.5p + .5q' \succcurlyeq .5p' + .5q \text{ if and only if } .5p + .5\widetilde{p}' \succcurlyeq .5p' + .5\widetilde{p}.$$

Observe that $\widetilde{p} \sim \widetilde{q}$, $\widetilde{p}' \sim \widetilde{q}'$ allows us to infer that the DM considers the difference between $\widetilde{p}$ and $\widetilde{p}'$ to be the same as that between $\widetilde{q}$ and $\widetilde{q}'$. At the same time, based on a similar argument as above, $.5q + .5\widetilde{q}' \sim .5q' + .5\widetilde{q}$ allows us to infer that DM considers the difference between $\widetilde{q}$ and $\widetilde{q}'$ to be the same as that between q and q'. Accordingly, on grounds

[13] In this case, $(\lambda p + (1-\lambda)p')_A = (\lambda q + (1-\lambda)q')_A$, $p_A = q_A$ and $p'_A = q'_A$.

of consistency, the axiom suggests that DM should consider the difference between p and p' to be at least as great as that between q and q', which $.5p + .5q' \succcurlyeq .5p' + .5q$ implies, if and only if she considers the difference between p and p' to be at least as great as that between $\widetilde{p}$ and $\widetilde{p}'$, which $.5p + .5\widetilde{p}' \succcurlyeq .5p' + .5\widetilde{p}$ implies.

Finally, consider allocation-lotteries in Δ in which we fix DM's outcome at a particular $z \in Z$. Then the only risk borne in such lotteries is by the other members of society. We shall denote generic allocation-lotteries of this type by a pair $([z],p_A)$, which specifies that DM gets $z \in Z$ for sure whereas the others' outcomes are determined according to the lottery $p_A \in \Delta(A)$. Our final axiom formalizes the idea that DM's own outcomes have a significant bearing on her evaluation of such allocation-lotteries.

AXIOM 5. Importance of Own Outcomes
If $([z],p_A)$, $([z],q_A) \in \Delta$ are such that $([z],p_A) \succ ([z],q_A)$, then there exists $\overline{z}$, $\underline{z} \in Z$ such that $([\overline{z}], q_A) \succcurlyeq ([z],p_A)$ and $([z],q_A) \succcurlyeq ([\underline{z}],p_A)$.

In words, suppose that when DM's outcome is held fixed at some $z \in Z$, she would much rather have the others bear the risk (specified by) p_A than (that specified by) q_A. What this axiom, then, requires is that there exists some ("good") outcome $\overline{z} \in Z$ for DM, such that she prefers the allocation-lottery, which gives her $\overline{z}$ for sure and makes the others bear the risk q_A, than the allocation-lottery, which gives her z for sure and makes the others bear the risk p_A. Similarly, there exists some ("bad") outcome $\underline{z} \in Z$ for DM, such that she prefers the allocation-lottery, which gives her z for sure and makes the others bear the risk q_A than the allocation-lottery, which gives her $\underline{z}$ for sure and makes the others bear the risk p_A.

3.2 Representation

We now formally define an AEU representation and establish that the axioms we have proposed so far constitute a choice-theoretic foundation for such a representation.

Definition 1. *An **adjusted expected utility (AEU)** representation of $\succcurlyeq$ consists of a pair of functions:*

- *$w : Z \times A \to \mathbb{R}$, which represents $\succcurlyeq$ restricted to $Z \times A$*
- *$g : \Delta(A) \to \mathbb{R}$, which satisfies $g([a]) = 0$ for all degenerate lotteries $[a] \in \Delta(A)$,*

such that the function W:$\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A),$$

represents $\succcurlyeq$. That is, for any p, $q \in \Delta$, $p \succcurlyeq q$ if and only if $W(p) \geq W(q)$.

Theorem 1. *Suppose importance of own outcomes holds. Then:*

1. *$\succcurlyeq$ on Δ satisfies weak order, Archimedean, opportunity-comparable independence and consistent evaluation of opportunities if and only if there exists an adjusted expected utility representation of $\succcurlyeq$.*

2. *If (w:$Z\times A\to\mathbb{R}$, g:$\Delta(A)\to\mathbb{R}$) and ($\widetilde{w}$:$Z\times A\to\mathbb{R}$, $\widetilde{g}$:$\Delta(A)\to\mathbb{R}$) are both AEU representations of $\succcurlyeq$, then there exists constants $\alpha > 0$, β, such that:*

$$\widetilde{w} = \alpha w + \beta \text{ and } \widetilde{g} = \alpha g.$$

As mentioned earlier, the function w represents DM's ranking of ex-post allocations. That is for any (z, a), $(z', a') \in Z\times A$, $[(z, a)] \succcurlyeq [(z', a')]$ if and only if $w(z, a) \geq w(z', a')$. Therefore, the first term, $\sum_{(z,a)\in Z\times A} p(z, a)w(z, a)$, captures DM's baseline expected utility evaluation of the lottery p. On the other hand, the function g represents the adjustment associated with any allocation-lottery, namely, it identifies by how much the baseline expected utility evaluation of a lottery needs to be adjusted or corrected to better reflect DM's assessment of the overall opportunities available to the others under that lottery. Observe that the essential uniqueness result (the second part of the theorem) implies that whereas the sign of the function w has no meaning, the sign of g does. The sign of the adjustment associated with any allocation-lottery, which can be either positive, negative or zero, is uniquely pinned down under an AEU representation. In particular, the adjustment associated with lotteries in which the others get some outcome for sure is zero.

4 Values, Tastes and Standards of Equity

In *Social Choice and Individual Values*, Kenneth Arrow suggests a distinction between a decision maker's tastes and her standards of equity, when it comes to evaluating social states or allocations:

> In general, there will, then be a difference between the ordering of social states according to the direct consumption of the individual and the ordering when the individual adds his general standards of equity. We may refer to the former ordering as reflecting the tastes of the individual and the latter as reflecting his values. The distinction between the two is by no means clear cut . . . no sharp line can be drawn between tastes and values (Arrow 1963).

Specializing Arrow's terminology to our revealed preference framework, we may say that what we take as an observable primitive in our model is a decision maker's *values*, that is, her choices over social allocations. As Arrow clarifies, these choices are potentially influenced by both her attitude towards her *direct* individual consumption, which we may call her *tastes*, as well as that towards the consumption of others, including her evaluation of their consumption relative to her's, which we may call her *standards of equity.* We may think of tastes as a purely private component in DM's evaluation of outcomes that is intrinsic to such an evaluation. On the other hand standards of equity constitutes a public or social component in DM's evaluation and may be thought of as extrinsic as far as the source of this evaluation is rooted in her existence in society. For instance, for Robinson Crusoe, marooned in an island all by himself, all that would matter in his evaluation of outcomes are his tastes.

In an AEU representation, what we identify is a function, $w{:}Z\times A \to \mathbb{R}$, over allocations, which reflects DM's values. However, this function does not allow us to disentangle DM's tastes from her standards of equity. In other words, for any set of allocations, this function keeps track of DM's preference ranking over the set, but it does not inform us as to what extent this ranking is influenced by her intrinsic concerns about her own outcomes, or her concerns about others' outcomes, including how she sees their consumption in relation to hers. Indeed, Arrow's observation that there may not be a clear cut distinction between tastes and values raises the question about whether it is at all possible to separate these two influences—tastes and standards of equity—just from observing DM's values and, if possible, what restrictions do such a separation imply on DM's choices. In this section we pursue the question of the possibility of separating tastes from standards of equity in an AEU set up.

4.1 Independent Risk Preference AEU Representation

One approach that we may take to the question of separating the decision maker's tastes from her standards of equity is to subsume it within the larger question of whether we can derive her risk preference over lotteries on her own outcomes—*own-lotteries*, for short—and her risk preference over lotteries on others' outcomes—*others'-lotteries*, for short—from observing her risk preference over allocation-lotteries. We formally introduce these concepts in the following definition. In the way of notation, note that for any allocation lottery $p \in \Delta$, $p_Z \in \Delta(Z)$ shall denote the marginal measure of p over Z.

Definition 2. *DM has independent risk preference over own-lotteries if for any p, q, p', q' $\in \Delta$ with $p_A = q_A$, $p'_A = q'_A$ and $p_Z = p'_Z$, $q_Z = q'_Z$,*

$$p \succcurlyeq q \text{ if and only if } p' \succcurlyeq q'.$$

Similarly, DM has independent risk preference over others'-lotteries if for any p, q, p', q' $\in \Delta$ with $p_Z = q_Z$, $p'_Z = q'_Z$ and $p_A = p'_A$, $q_A = q'_A$,

$$p \succcurlyeq q \text{ if and only if } p' \succcurlyeq q'.$$

In other words, DM has independent risk preference over own-lotteries if whenever any two allocation-lotteries generate the same marginal others'-lottery, her ranking of these allocation-lotteries is based solely on comparing the marginal own-lotteries under them, independent of what the common marginal others'-lottery happens to be. Similarly, she has independent risk preference over others'-lotteries if whenever any two allocation-lotteries generate the same marginal own-lottery, her ranking of these allocation-lotteries is based solely on comparing the marginal others'-lotteries under them, independent of what the common marginal own-lottery happens to be. Observe that if DM has independent risk preference over own-lotteries, we can elicit her ranking of own-lotteries based on her ranking of allocation-lotteries. In particular, we can define a preference relation $\widehat{\succcurlyeq}$ on $\Delta(Z)$ as follows:

$$p_Z \mathrel{\widehat{\succcurlyeq}} q_Z \text{ iff } p \succcurlyeq q, \text{ for any } p,\, q \in \Delta \text{ with } p_A = q_A.$$

In a similar vein, if DM has independent risk preference over others'-lotteries, we can elicit her ranking over others'-lotteries based on her ranking of allocation-lotteries.

The following definition suggests a natural way of incorporating independent risk preference over own-lotteries as well as others'-lotteries in an AEU representation.

Definition 3. *An* ***independent risk preference*** *AEU representation of $\succcurlyeq$ is a collection of functions,*

- $u : Z \to \mathbb{R}$,
- $v : A \to \mathbb{R}$, *and*
- $g : \Delta(A) \to \mathbb{R}$, *with $g([a]) = 0$ for all degenerate lotteries $[a] \in \Delta(A)$,*

such that the function $W{:}\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{z\in Z} p_Z(z)u(z) + \sum_{a\in A} p_A(a)v(a) + g(p_A),$$

represents $\succcurlyeq$. That is, for any p, $q \in \Delta$, $p \succcurlyeq q$ if and only if $W(p) \geq W(q)$.

In other words, under an independent risk preference AEU representation, DM's evaluation of any allocation-lottery $p \in \Delta$ is given by the sum of three terms—the expected utility evaluation of the marginal own-lottery p_Z, the expected utility evaluation of the marginal others'-lottery p_A, and the adjustment. It is straightforward to verify that under this representation, DM has independent risk-preference over her own-lotteries as well as others'-lotteries.

We now identify a condition on DM's choice behavior, which, when added to the earlier axioms, allows us to represent her preferences with an independent risk preference AEU representation. This condition essentially requires DM's standards of equity to be independent of the outcomes that she herself gets in a fairly strong sense.

AXIOM 6. Strong Independent Standards of Equity
Suppose $[(z,a)]$, $[(z',a')]$, p, $p' \in \Delta$ *such that* $p_A = p'_A$ *and* $p \sim [(z,a)] \succ [(z',a')] \sim p'$. *Then for any* $\lambda \in [0,1]$,

$$[(z,a')] \succcurlyeq \lambda p + (1-\lambda)p' \text{ if and only if } \lambda p' + (1-\lambda)p \succcurlyeq [(z',a)].$$

Strong independent standards of equity formalizes the idea that the improvement (or reduction) in DM's welfare in moving from the allocation (z,a') to (z,a) should be of the "same magnitude" as that when she moves from the allocation (z',a') to (z',a). In other words, the particular outcome that she gets—z or z'—should not influence this evaluation. The following result establishes the connection between strong independent standards of equity and an independent risk preference AEU representation.

Theorem 2. *Suppose importance of own outcomes holds. Then:*

1. $\succcurlyeq$ *on* Δ *satisfies weak order, Archimedean, opportunity-comparable independence, consistent evaluation of opportunities and strong independent standards of equity if and only if there exists an independent risk preference AEU representation of* $\succcurlyeq$.

2. *If* $(u{:}Z\to\mathbb{R},\ v{:}A\to\mathbb{R},\ g{:}\Delta(A)\to\mathbb{R})$ *and* $(\widetilde{u}{:}Z\to\mathbb{R},\ \widetilde{v}{:}A\to\mathbb{R},\ \widetilde{g}{:}\Delta(A)\to\mathbb{R})$ *are both independent risk preference AEU representations of* $\succcurlyeq$, *then there exists constants* $\alpha > 0$, β', β'' *such that:*

$$\widetilde{u} = \alpha u + \beta',\ \widetilde{v} = \alpha v + \beta'' \text{ and } \widetilde{g} = \alpha g.$$

4.2 Taste-Equity Separable AEU Representation

The independent risk preference AEU representation provides a sharp characterization of DM's risk preferences over both her own-lotteries and others'-lotteries. In so doing, it allows us to draw a strong separation between her tastes and standards of equity within a revealed preference framework. However, as Fudenberg and Levine (2011)—F-L hereafter—point out, the possibility of making this strong separation comes at a significant cost. This representation cannot accommodate what they refer to as "ex-post fairness." The content of ex-post fairness lies in the observation that, in evaluating an allocation-lottery, a decision maker may care not just about the own-lottery and others'-lottery generated by it, but also about the correlation between her outcomes and those of others. To appreciate this point better, consider the following example that F-L suggest. Suppose that a decision maker has to choose between the following two allocation-lotteries, p and q, in a two-person world. Under p with even chances the allocation is either $(\$100,\$100)$ or $(\$0,\$0)$, whereas under q with even chances the allocation is either $(\$100,\$0)$ or $(\$0,\$100)$. Note that a decision maker, whose preferences can be represented by an independent risk preference AEU representation, considers p and q to be indifferent. On the other hand, as F-L argue, it is not hard to imagine, that there may be a significant proportion of decision makers who strictly prefer p to q on the grounds that p results in fairer ex-post allocations.

One way of understanding why an independent risk preference AEU representation cannot accommodate the plausible choices suggested by F-L is in terms of the restriction imposed on DM's choices by the condition of strong independent standards of equity. To see this, consider a pro-social decision maker who would strictly prefer that the other person receives \$100 rather than \$0 whenever so doing imposes no costs on her, in particular, when her outcomes are held fixed. What strong independent standards of equity rules out is the possibility that the magnitude of her welfare gain from seeing the other person getting \$100 instead of \$0 may depend on the particular outcome she herself receives. For example, it may not be unreasonable to imagine that this welfare gain is much higher when she herself receives \$100 than when she receives \$0—a possibility that this condition does not allow. What this discussion, therefore, alludes to is the fact that the strength or intensity of a decision maker's concerns for equity may be related to what her personal outcomes are. It is with a goal of formalizing this intuition that we suggest the following AEU representation.

Definition 4. *Fix $z^* \in Z$ and $a^* \in A$. A* ***taste-equity separable*** *AEU representation of $\succcurlyeq$ is a collection of functions,*

- $u : Z \to \mathbb{R}$,
- $v : A \to \mathbb{R}$,
- $\theta : Z \to \mathbb{R}_{++}$, *with* $\theta(z^*) = 1$ *and*
- $g : \Delta(A) \to \mathbb{R}$, *with* $g([a]) = 0$ *for all degenerate lotteries* $[a] \in \Delta(A)$,

such that the function $W : \Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{u(z) + \theta(z)(v(a) - v(a^*))\} + g(p_A),$$

represents $\succcurlyeq$. That is, for any p, $q \in \Delta$, $p \succcurlyeq q$ if and only if $W(p) \geq W(q)$.

A taste-equity separable AEU representation identifies two real valued functions, u and θ, over DM's outcomes and a real valued function, v, over others' outcomes, in addition to the adjustment, g. Note that, under this representation, for any z, $z' \in Z$, a, $a' \in A$,

$$[(z,a)] \succcurlyeq [(z,a')] \Leftrightarrow v(a) \geq v(a') \Leftrightarrow [(z',a)] \succcurlyeq [(z',a')]$$

In other words, DM's ranking of others' outcomes is independent of whatever outcome she receives and the function v represents this ranking. Note that in evaluating any outcome, $a \in A$, that others get under any allocation, DM considers the difference $v(a) - v(a^*)$. We may interpret $a^* \in A$ as a reference point that DM has when it comes to the outcome of others, and accordingly $v(a^*)$ as a reference payoff for the others. Next, note that associated with any outcome $z \in Z$, which the decision maker may receive, is a positive constant $\theta(z)$, which measures the intensity or strength of DM's concern for others' outcomes when she receives the outcome z. We fix a particular outcome $z^* \in Z$ for which $\theta(z^*)$ is normalized to 1. Under this representation, the outcome that DM gets influences her evaluation of an allocation in two ways—first, through the function u and second, through the function θ. However, when the others' outcome is a^*, the second consideration drops out. That is, for any z, $z' \in Z$,

$$[(z, a^*)] \succcurlyeq [(z', a^*)] \Leftrightarrow u(z) \geq u(z').$$

In other words, when the others' outcome is held fixed at a^*, DM's ranking of allocations in which only her outcome is varied, provides us with a direct ranking of how she values her outcomes from a "private" perspective, abstracting from social considerations. It is this private ranking that we think of as her tastes and, therefore, the function u represents her tastes.

Observe that the choices of the decision maker of F-L's example can indeed be accommodated by a tastes-equity separable AEU representation. A decision maker whose choices can be represented thus and for whom $v(100) > v(0)$ will choose the lottery p of the example over the lottery q if $\theta(100) > \theta(0)$, that is, if the intensity of her concerns for others' outcomes are more pronounced when she receives \$100 than when she receives \$0.

We now introduce a condition that is weaker than strong independent standards of equity, and show that if DM's choices satisfy this weaker condition, along with the axioms required for an AEU representation, then her preferences can indeed be represented by a taste-equity separable AEU representation.

AXIOM 7. Weak Independent Standards of Equity
Suppose z, $z' \in Z$, a, $a' \in A$, p, p', q, $q' \in \Delta$, with $p_A = p'_A$, $q_A = q'_A$, are such that $[(z, a)] \sim p$, $[(z, a')] \sim p'$, $[(z', a)] \sim q$, $[(z', a')] \sim q'$. Then for any $a'' \in A$ and $\lambda \in [0, 1]$,

$$[(z, a'')] \succcurlyeq \lambda p + (1 - \lambda)p' \textit{ if and only if } [(z', a'')] \succcurlyeq \lambda q + (1 - \lambda)q'.$$

Weak independent standards of equity formalizes the idea that if, say for instance, $[(z, a)] \succ [(z, a'')] \succ [(z, a')]$, then not only is this ranking maintained when z is replaced with z' but also the relative position of (z, a'') in relation to (z, a) and (z, a') on DM's subjective preference scale is the same as that of (z', a'') in relation to (z', a) and (z', a'). For instance, if DM considers moving from the allocation (z, a') to (z, a'') to be equally as good as moving from (z, a'') to (z, a), then she considers moving from (z', a') to (z', a'') to be equally as good as moving from (z', a'') to (z', a). The following result establishes the connection between this axiom and a taste-equity separable AEU representation.

Theorem 3. *Suppose importance of own outcomes holds. Then:*

1. *$\succcurlyeq$ on Δ satisfies weak order, Archimedean, opportunity-comparable independence, consistent evaluation of opportunities and weak independent standards of equity if and only if there exists a taste-equity separable AEU representation of $\succcurlyeq$.*

2. *If (u: $Z \to \mathbb{R}$, θ: $Z \to \mathbb{R}_{++}$, v: $A \to \mathbb{R}$, g: $\Delta(A) \to \mathbb{R}$) and ($\widetilde{u}$: $Z \to \mathbb{R}$, $\widetilde{\theta}$: $Z \to \mathbb{R}_{++}$, $\widetilde{v}$: $A \to \mathbb{R}$, $\widetilde{g}$: $\Delta(A) \to \mathbb{R}$) are both taste-equity separable AEU representations of $\succcurlyeq$, and $(z', a') \succ (z', a'')$ for some $z' \in Z$, a', $a'' \in A$, then there exists constants $\alpha > 0$, β and β' such that:*

$$\widetilde{u} = \alpha u + \beta,\ \widetilde{v} = \alpha v + \beta',\ \widetilde{\theta} = \theta \text{ and } \widetilde{g} = \alpha g$$

Consider a taste-equity separable adjusted expected utility representation (u:$Z \to \mathbb{R}$, θ:$Z \to \mathbb{R}_{++}$, v:$A \to \mathbb{R}$, g:$\Delta(A) \to \mathbb{R}$). The essential uniqueness result tells us that the decision maker's tastes, represented by the function u, and her ranking of others' outcomes represented by the function v, can be identified essentially uniquely. Further, the strength of DM's concern about others' outcomes, represented by the function θ, is uniquely identified under such a representation.

Before concluding this section, one question that we need to address is about the possibility of representing DM's preferences by a function W:$\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{(z,a) \in Z \times A} p(z, a)\{u(z) + \theta(z)v(a)\} + g(p_A),$$

where the functions u, v, θ and g have similar interpretations as under a taste-equity separable AEU representation. As it turns out, the axioms listed in Theorem 3 are indeed sufficient to derive such a representation. However, in such a representation, whereas the function v representing DM's ranking of others' outcomes, the function θ representing the strength of DM's concerns for others' outcomes and the function g representing the adjustment can all be uniquely identified along the same lines as in the above theorem, DM's tastes, which we represent by the function u, can no longer be uniquely identified. It can be shown that if ($\widetilde{u}$:$Z \to \mathbb{R}$, $\widetilde{\theta}$:$Z \to \mathbb{R}_{++}$, $\widetilde{v}$:$A \to \mathbb{R}$, $\widetilde{g}$:$\Delta(A) \to \mathbb{R}$) is another representation of the above type, that is, if the function $\widetilde{W}$:$\Delta \to \mathbb{R}$, given by

$$\widetilde{W}(p) = \textstyle\sum_{(z,a) \in Z \times A} p(z, a)\{\widetilde{u}(z) + \widetilde{\theta}(z)\widetilde{v}(a)\} + \widetilde{g}(p_A),$$

represents $\succcurlyeq$ as well, then there exists constants $\alpha > 0$, $\sigma > 0$, μ, β such that:

1. $\widetilde{u}(z) - \widetilde{u}(z') = \alpha(u(z) - u(z')) - \frac{\mu\alpha}{\sigma}(\theta(z) - \theta(z'))$, for all z, $z' \in Z$,
2. $\widetilde{v}(a) = \sigma v(a) + \mu$, for all $a \in A$,
3. $\widetilde{\theta}(z) = \theta(z)$, for all $z \in Z$ and
4. $\widetilde{g}(p_A) = \alpha g(p_A)$, for any $p_A \in \Delta(A)$.

In other words, DM's ranking of two outcomes z, $z' \in Z$ according to the function u, representing her tastes, will be preserved under the function $\widetilde{u}$ if the intensity factors $\theta(z)$ and $\theta(z')$ are the same but not necessarily otherwise.

5 A Conjecture on the Adjustments

In the analysis so far, we have shown that it is possible to identify from choice data the adjustment that a decision maker may make to her expected utility evaluation of a lottery to account for her concerns about the ex-ante opportunities available to the others under that lottery. We have, however, not provided a precise specification of what exact form such an adjustment may take. In this section we propose a specific formulation for how the decision maker makes such adjustments as well as show how this formulation can be identified from choice data. Our formulation relies on a basic asymmetry. Under it, a decision maker makes an upward adjustment to her expected utility evaluation of a lottery *only* in the events where, by her subjective evaluation, she considers the outcome that the others receive to be worse than their "average outcome" under that lottery. In other words, in these events DM considers the payoff associated with the expected utility evaluation of the lottery to not accurately reflect the overall opportunities that are available to the others under that lottery. To put things more precisely, observe that both under the independent risk preference and taste-equity separable AEU representations, we identify a function, $v : A \to \mathbb{R}$, that represents DM's ranking of others' outcomes. Under our proposed formulation, which we refer to as *asymmetric adjustment*, DM uses this function v to determine the adjustment associated with an allocation-lottery $p \in \Delta$, which generates a marginal others'-lottery, $p_A = [a^1, \lambda^1; \ldots ; a^N, \lambda^N] \in \Delta(A)$, in the following manner:

$$g(p_A) = \textstyle\sum_{n=1}^{N} \lambda_n.\max\{\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}}) - v(a^n), 0\}.$$

Accordingly, under this specification of the adjustment DM considers the expected payoff from p_A, given by $\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}})$, to be indicative of the overall opportunities available to the others under the lottery p. Whenever, the realized outcome for the others, say a^n, is such that $v(a^n)$ is less than this expected payoff, $\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}})$, she makes an upward adjustment to the expected utility evaluation that is equal to the difference of $\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}})$ and $v(a^n)$. No adjustment is made when the realized outcome for the others, a^n, is such that $v(a^n)$ is at least as great as $\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}})$. The total adjustment is then determined by the probability-weighted average of these individual adjustments.

We next provide a choice-theoretic characterization of asymmetric adjustments. To do so, we need to introduce the following definition.

Definition 5. $(\underline{\lambda},\ \underline{p},\ \overline{p}) \in [0,1] \times \Delta \times \Delta$ *is a* ***bad-good decomposition*** *of* $p = ([z],\ p_A)$ $= [(z,\ a^1),\ \lambda_1;\ .\ .\ .\ ;\ (z,\ a^N),\ \lambda_N] \in \Delta$ *if* $p = \underline{\lambda}\underline{p} + (1 - \underline{\lambda})\overline{p}$ *and there exists* $q^1,\ .\ .\ .\ ,\ q^N$ $\in \Delta$ *with* $q^1_A = q^2_A = .\ .\ . = q^N_A$ *and* $[(z,\ a^n)] \sim q^n$ *for all* $n = 1,\ .\ .\ .\ ,\ N$ *such that* $(z,\ a^n)$ *is in the support of* $\overline{p}$ *if and only if* $[(z,\ a^n)] \sim q^n \succcurlyeq \lambda_1 q^1 + .\ .\ . + \lambda_N q^N$.
Further, under this decomposition we will refer to the lottery $\sum_{\{n:(z,a^n)\in supp(\underline{p})\}}(\lambda_n/\underline{\lambda})q^n$ *as the* ***bad outcomes equivalent*** *of* p *and denote it by* $\underline{p}^*$*, and the lottery* $\sum_{\{n:(z,a^n)\in supp(\overline{p})\}}(\lambda_n/1-\underline{\lambda})q^n$ *as the* ***good outcomes equivalent*** *of* p *and denote it by* $\overline{p}^*$.

The idea behind a bad-good decomposition of a lottery $p = ([z],\ p_A)$ is to split it up into two sub-lotteries, $\underline{p}$ and $\overline{p}$, such that one of the sub-lotteries, $\underline{p}$, contains only the "bad" or "below-average" outcomes under the lottery and the other, $\overline{p}$, contains only the "good" or "above-average" outcomes. Of course, it is not immediate how we should ascertain what the average outcome under the lottery p is. In the definition, this is sought to be defined through the allocation-lotteries $q^1,\ .\ .\ .\ ,\ q^N$ in Δ. Observe that from a preference perspective, for each n, the lottery q^n is equivalent to the allocation $(z,\ a^n)$. Further, since each of the lotteries $q^1,\ .\ .\ .\ ,\ q^N$ provide the others with the same ex-ante opportunities and so, by opportunity-comparable independence, they can be evaluated separably, the compound lottery $\lambda_1 q^1 + .\ .\ . + \lambda_N q^N$ can be considered, from a preference perspective, to be equivalent to the average outcome under the lottery p. Hence, if an allocation $(z,\ a^n)$ in the support of p is at least as good as $\lambda_1 q^1 + .\ .\ . + \lambda_N q^N$, it is considered a good outcome, else it is considered bad. Once we have split up the support of p in this manner into good and bad outcomes, we use this division to define the good outcome equivalent, $\overline{p}^*$, and bad outcomes equivalent, $\underline{p}^*$, of p.

We will now make use of a bad-good decomposition to state an axiom, which characterizes asymmetric adjustment. The specification of the axiom is slightly different depending on whether we are considering an independent risk preference or a taste-equity separable AEU representation. For the former the following suffices.

AXIOM 8. Overweighing
If $(\underline{\lambda},\ \underline{p},\ \overline{p}) \in [0,1] \times \Delta \times \Delta$ *is a bad-good decomposition of* $p = ([z],\ p_A) \in \Delta$ *and* $\underline{p}^*$, $\overline{p}^*$ *are respectively the bad and good outcomes equivalent under this decomposition, then*

$$p \succcurlyeq \lambda \underline{p}^* + (1 - \lambda)\overline{p}^* \text{ if and only if } \lambda \geq \underline{\lambda}^2.$$

On the other hand, for a taste-equity separable AEU representation, we need the axiom to apply only for bad-good decomposition of allocation-lotteries of the type $([z], p_A) \in \Delta$ for a specific outcome z. Recall that under a taste-equity separable AEU representation ($u{:}Z{\to}\mathbb{R}$, $\theta{:}Z \to\mathbb{R}_{++}$, $v{:}A{\to}\mathbb{R}$, $g{:}\Delta(A){\to}\mathbb{R}$) of $\succcurlyeq$, there exists an outcome for DM, $z^* \in Z$, for which $\theta(z^*) = 1$, i.e., the intensity factor associated with z^* is normalized to 1. It is in relation to this specific z^* that we need the axiom to hold in this case.

AXIOM 9. Overweighing*
If $(\underline{\lambda}, \underline{p}, \overline{p}) \in [0,1] \times \Delta \times \Delta$ is a bad-good decomposition of $p = ([z^*], p_A) \in \Delta$ and $\underline{p}^*$, $\overline{p}^*$ are respectively the bad and good outcomes equivalent under this decomposition, then

$$p \succcurlyeq \lambda \underline{p}^* + (1 - \lambda)\overline{p}^* \text{ if and only if } \lambda \geq \underline{\lambda}^2.$$

As we have argued above, from a preference perspective, $\underline{\lambda}\underline{p}^* + (1 - \underline{\lambda})\overline{p}^*$ is equivalent to the (probability-weighted) average outcome under the lottery p. So, if DM were an expected utility maximizer, she would consider the lottery p to be indifferent to the lottery $\underline{\lambda}\underline{p}^* + (1 - \underline{\lambda})\overline{p}^*$. However, decision makers that we are modeling may care about the ex-ante opportunities available to the others as well. This axiom captures how this concern influences DM's choices. In particular, the axiom says that DM evaluates the lottery p as if she were overweighing the good outcomes under it and underweighing the bad ones compared to how they are weighed under an expected utility evaluation. Given that the reason for DM's departure from an expected utility evaluation is her concern for others' overall opportunities and that her own outcomes are held fixed under the lottery p, this axiom, accordingly, implies that DM's perception of others' overall opportunities is swayed by the good outcomes that she deems others get under a lottery. This axiom characterizes asymmetric adjustments.

Proposition 1. *Suppose $g{:}\Delta(A) \to \mathbb{R}$ is the adjustment under an independent risk preference (respectively, a taste-equity separable) AEU representation of $\succcurlyeq$. Then $\succcurlyeq$ satisfies overweighing (respectively, overweighing*) if and only if*

$$g(p_A) = \textstyle\sum_{n=1}^{N} \lambda_n.\max\{\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}}) - v(a^n), 0\}.$$

It is worth noting that under asymmetric adjustment, the adjustment associated with any allocation-lottery is always non-negative. The reason for this is the asymmetry that exists in the formulation of this adjustment. In particular, DM makes upward adjustments only in events where she considers the outcome that others receive to be worse than what they receive on average, but no adjustment is made otherwise. In order to gain more intuition on how this particular specification of the adjustment works we next consider an example involving the probabilistic dictator game and see what choices are implied by our model in this context and how they match up with some evidence that we have from a recent set of experiments conducted by Krawzyk and Lelec (2010).

5.1 Example: Probabilistic Dictator Game

In a standard two person dictator game one of the persons, called the dictator, is endowed with a fixed amount of money. She can then decide whether she wants to keep the entire amount to herself or give some of it to the other person. As opposed to this, in a probabilistic dictator game, the dictator is not allowed to share the money with the other person, but if she chooses she may assign some probability to the other person getting the entire amount of money, while retaining the complimentary probability of getting the entire amount herself. A recent set of experiments run by Krawzyk and LeLec show that in this setting a significant portion of decision makers do indeed assign some probability—on average about 10% or so—to the other person getting the entire amount. The reasoning behind these decision makers' choices does not seem hard to fathom—presumably all that they are trying to do is compensate for the lack of ex-post fairness which is inevitable in this environment by sharing ex-ante chances more fairly. However, from the perspective of standard models of decision making under risk these choices, like those of Beth, appear puzzling as they violate stochastic dominance. To see this, consider a decision maker who strictly prefers the allocation in which she gets the entire amount of money to the allocation in which the other person gets the entire amount. For such a decision maker, stochastic dominance requires that she should never give any chance to the other person getting the money in the probabilistic dictator game.[14] Given the failure of standard models of decision making under risk to explain these choices, it is instructive to see what choices are implied by the AEU model. For this analysis, we use an independent risk preference AEU representation along with an asymmetric adjustment.

Consider a decision maker who is faced with the problem of deciding what probability $\lambda \in [0, 1]$ she wants to assign to the other person getting the entire amount of money—suppose it is \$20—in the probabilistic dictator game discussed above. For any choice of λ, we get an allocation-lottery,

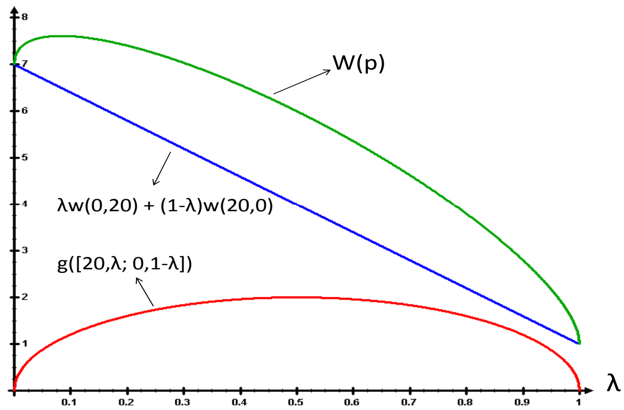
$$p = [(0, 20),\ \lambda;\ (20, 0),\ 1 - \lambda],$$

under which the allocation $(0, 20)$ realizes with probability λ, and the allocation $(20, 0)$ realizes with probability $1 - \lambda$. Under an independent risk preference AEU representation with asymmetric adjustment, DM evaluates a lottery like p by the following function:

$$W(p) = \lambda w(0, 20) + (1 - \lambda)w(20, 0) + g([20,\ \lambda;\ 0,\ 1 - \lambda]),$$

[14]A similar conclusion about not sharing chances also follows if we consider the less plausible case of a decision maker who strictly prefers the allocation in which the other person gets the entire sum of money to the allocation in which she gets the entire sum.

Figure 1: Choices in the Probabilistic Dictator Game.



where,

$$w(0,20) = u(0) + v(20), w(20,0) = u(20) + v(0)$$
$$g([20,\lambda;0,1-\lambda]) = (1-\lambda)(\lambda v(20) + (1-\lambda)v(0) - v(0))$$

Accordingly,

$$W(p) = u(20) + v(0) - \lambda[u(20) - u(0) - (v(20) - v(0))] + \lambda(1-\lambda)(v(20) - v(0))$$

Observe that λ appears both in the expected utility evaluation as well as the adjustment. It is straightforward to verify that the expected utility payoffs decreases linearly in λ (since $w(20,0) > w(0,20)$). On the other hand, the adjustment is concave in λ, with it first increasing, attaining a maximum at $\lambda = \frac{1}{2}$, and then decreasing. Accordingly, as λ increases from 0 to something slightly larger, a tradeoff emerges. By the expected utility component, as λ increases, she gets worse off, but by her concerns for the other person's ex-ante opportunities, represented by the adjustment, she is better off. The overall payoff is determined by the interaction of these two opposing influences. In particular, note that if

$$v(20) - v(0) > u(20) - u(0) - (v(20) - v(0)) \Leftrightarrow v(20) - v(0) > 0.5(u(20) - u(0)),$$

then for small values of λ, on increasing λ slightly, the incremental improvement in the adjustment outweighs the drop-off in the expected utility payoffs. That is why under our representation the payoff $W(p)$ from a lottery like $p = [(0,20),\ \lambda;\ (20,0),\ 1-\lambda]$ may be increasing in λ for small values of λ, and hence, DM may strictly prefer giving the other person some chance of getting the money, even though she strictly prefers the allocation in which she gets the money, namely $(20,0)$, to the allocation in which the other person receives the money, namely, $(0,20)$.

6 Other-Regarding Preferences and Consequentialism: Comments on the Literature

This paper draws a connection between other-regarding preferences and nonseparable preferences in environments of risk. A decision maker's preferences are *nonseparable* across mutually exclusive events if her evaluation of an outcome in a given event is intrinsically tied to considerations relating to other events that could have occurred but did not. In the words of Machina (1989), "An agent with nonseparable preferences feels (both ex ante and ex post) that risk which is borne but not realized is gone in the sense of having been *consumed* (or "borne"), rather than gone in the sense of *irrelevant*."[15] These are decision makers who violate the *independence* axiom of expected utility theory. It is worth highlighting here that the condition of independence/event-separability can be derived from another, more fundamental, normative principle of behavior. Hammond (1988) shows that decision makers whose preferences conform with independence satisfy the property of 'consequentialism'; that is, their behavior is entirely explicable by its consequences. Informally, this says that a decision maker's preference for some 'sub-lottery' p over some other 'sub-lottery' q does not depend in any way on the form or content of 'parent lotteries,' in which p and q may be embedded.[16]

There is a large decision theoretic literature that accommodates preferences that are nonseparable and hence violate the independence axiom. Prominent examples include rank dependent utility, *betweenness* based theories (like implicit expected utility) and generalized prospect theory.[17] The key feature of these models is that although event-separability of preferences is not required to hold on the space of all lotteries, each of them identifies a subset of lotteries over which preferences are separable (see Chew and Epstein (1988) for an illustration of this point). Decision makers whose preferences are accommodated by any of these models may then be thought to conform with consequentialism in a 'restricted sense.' In particular, under all these models behavior retains the following *minimal notion* of consequentialism: Suppose x and y are elements in some underlying set of outcomes and p is a lottery on that set. Then if the decision maker prefers x to y, she must prefer the compound lottery that gives x with some positive probability $\lambda > 0$, and p with probability $1 - \lambda$ to the compound lottery that gives y with probability λ and p with probability $1 - \lambda$. This

[15] The emphases in the quote are as in the original.

[16] More formally, let X be a set of outcomes and $\Delta(X)$ the space of lotteries defined on X. Let Ξ be a 'rich class of decision problems.' Let $F : \Xi \rightrightarrows \Delta(X)$ denote a feasibility correspondence that specifies the feasible set of lotteries for any decision problem $\xi \in \Xi$. Let $B : \Xi \rightrightarrows \Delta(X)$ be a behavior correspondence that specifies the decision maker's choice behavior in any decision problem. The decision maker is a consequentialist if there exists a choice correspondence $C : 2^{\Delta(X)} \setminus \emptyset \rightrightarrows \Delta(X)$ such that for all $\xi \in \Xi$, $B(\xi) = C(F(\xi))$. In other words, changes in the structure of a decision problem should have no bearing on choices unless they change the feasible set. It is critical to recognize that we provide this definition for a *given* set of outcomes that we hold fixed.

[17] Refer to Starmer (2000) for a comprehensive survey of non-expected utility models.

condition, which has been called the *axiom of degenerate independence* (ADI) by Grant, Kajii and Polak (1992), implies stochastic dominance as well as the implication that the decision maker has a ranking over outcomes that is independent of the particular lottery in which such outcomes are realized. In contrast, in the decision model of nonseparable preferences that we present in this paper, we do not require behavior to conform to this minimal notion of consequentialism. As we have argued, consequentialism, even in this minimalist sense, rules out very plausible patterns of choice behavior of decision makers with other-regarding preferences.

The important message, then, that our paper seeks to bring out is that in environments featuring risk there may be a very basic conflict between the notions of separability of preferences, which implies a consequentialist evaluation of prospects, and other-regarding preferences, which, as we have seen, may incorporate a concern for the process by which outcomes are determined. In very simple terms, this conflict arises because decision makers with other-regarding preferences may not have an *absolute* yardstick with which to evaluate social allocations. Rather such evaluations may be made *relative* to the overall context in which they deem end-outcomes have been determined. In our opinion, this critical link between between other-regarding preferences and non-separable/non-consequentialist preferences has not been systematically pursued in the literature. It is our hope that future work shall correct for this.

7 Appendix

7.1 Proof of Theorem 1

The proof of sufficiency of the axioms for the representation for the case $\succ = \emptyset$ is immediate: simply set $w(z,a)$ equal to a constant for all $(z,a) \in Z \times A$ and g to be identically equal to 0. We consider here the proof of sufficiency for the case when $\succ \neq \emptyset$. The proof proceeds through several lemmas. We begin with a lemma that establishes that the preference domain has "no holes"—formally, that it is connected with respect to the order topology generated by $\succcurlyeq$. To state it, consider the following notation. For any $p_A \in \Delta(A)$, define

$$\begin{aligned} \Delta(p_A) &= \{q \in \Delta : q_A = p_A\}, \text{ and} \\ \Delta^*(p_A) &= \{q \in \Delta : q' \succ q \succ q'', \text{ for some } q', q'' \in \Delta(p_A)\} \end{aligned}$$

Note that by **opportunity-comparable independence**, it follows that $\Delta^*(p_A) \neq \emptyset$ if there exists p, $p' \in \Delta(p_A)$ with $p \succ p'$.

Lemma 1. *For any p, $q \in \Delta$, if $p \succ q$, then exactly one of the following three cases hold.*

1. $\Delta^*(p_A) = \emptyset$, $\Delta^*(q_A) \neq \emptyset$ *and there exists* $q^* \in \Delta(q_A)$ *such that* $q^* \sim p$.

2. $\Delta^*(q_A) = \emptyset$, $\Delta^*(p_A) \neq \emptyset$ *and there exists* $p^* \in \Delta(p_A)$ *such that* $p^* \sim q$.

3. $\Delta^*(p_A) \cap \Delta^*(q_A) \neq \emptyset$.

Proof. Suppose $\Delta^*(p_A) = \emptyset$. It follows that $([z], p_A) \sim p$ for all $z \in Z$. Consider any $z' \in Z$. There are two possibilities. The first possibility is that $([z'], q_A) \succcurlyeq ([z'], p_A)$. The second possibility is that $([z'], p_A) \succ ([z'], q_A)$. In this case, by **importance of own outcomes**, there exists $\overline{z} \in Z$ such that $([\overline{z}], q_A) \succcurlyeq ([z'], p_A)$. Therefore, under each of the possibilities, it follows there exists $q' \in \Delta(q_A)$ satisfying $q' \succcurlyeq p \succ q$. Hence, $\Delta^*(q_A) \neq \emptyset$. Further, by the **Archimedean** condition and **opportunity-comparable independence** there exists $q^* \in \Delta(q_A)$ such that $q^* \sim p$. By a similar argument, it follows that if $\Delta^*(q_A) = \emptyset$, then $\Delta^*(p_A) \neq \emptyset$ and there exists $p^* \in \Delta(p_A)$ such that $p^* \sim q$.

Next, consider the case when both $\Delta^*(p_A)$ and $\Delta^*(q_A)$ are non-empty. In order to prove that $\Delta^*(p_A) \cap \Delta^*(q_A) \neq \emptyset$ it suffices to show that either of the following conditions hold:

- (A) There exists $p' \in \Delta(p_A)$, q', $q'' \in \Delta(q_A)$ such that $q' \succ p' \succ q''$, or
- (B) There exists $q' \in \Delta(q_A)$, p', $p'' \in \Delta(p_A)$ such that $p' \succ q' \succ p''$.

In order to show that either (A) or (B) holds, we first establish that there exists z, $z' \in Z$ and a in the support of p_A such that $[(z, a)] \succ [(z', a)]$. To see this, note that since $\Delta^*(p_A) \neq \emptyset$, there exists $\tilde{p}$, $\tilde{p}' \in \Delta(p_A)$ such that $\tilde{p}$ and $\tilde{p}'$ are not indifferent. Before proceeding further, we introduce some new notation. We denote the lottery p_A by $[a_1, \lambda_1; \ldots ; a_N, \lambda_N]$. For any $n = 1, \ldots, N$, we let $\tilde{p}_{Z,a_n}$ (respectively, $\tilde{p}'_{Z,a_n}$) denote the conditional measure of $\tilde{p}$ (respectively, $\tilde{p}'$) on Z when others get the outcome a_n. Further, for any $n = 1, \ldots, N$, we let $(\tilde{p}_{Z,a_n}, [a_n])$ (respectively, $(\tilde{p}'_{Z,a_n}, [a_n])$) denote the allocation-lottery in Δ under which others get a_n for sure and DM's outcomes are determined by the lottery $\tilde{p}_{Z,a_n}$ (respectively, $\tilde{p}'_{Z,a_n}$). We may then express the two lotteries $\tilde{p}$ and $\tilde{p}'$ as:

$$\begin{aligned} \tilde{p} &= \lambda_1(\tilde{p}_{Z,a_1}, [a_1]) + \ldots + \lambda_N(\tilde{p}_{Z,a_N}, [a_N]) \\ \tilde{p}' &= \lambda_1(\tilde{p}'_{Z,a_1}, [a_1]) + \ldots + \lambda_N(\tilde{p}'_{Z,a_N}, [a_N]) \end{aligned}$$

Note that there must exist a_n for some $n = 1, \ldots, N$ and $\tilde{z}$, $\tilde{z}'$ in the support of $\tilde{p}_{Z,a_n}$ and $\tilde{p}'_{Z,a_n}$ respectively such that $[(\tilde{z}, a_n)]$ and $[(\tilde{z}', a_n)]$ are not indifferent. To see why, observe that if $[(\tilde{z}, a_n)] \sim [(\tilde{z}', a_n)]$ for all $\tilde{z}$ in the support of $\tilde{p}_{Z,a_n}$ and $\tilde{z}'$ in the support $\tilde{p}'_{Z,a_n}$, then by **opportunity-comparable independence**, it follows that $(\tilde{p}_{Z,a_n}, [a_n]) \sim (\tilde{p}'_{Z,a_n}, [a_n])$. If this were true for every $n = 1, 2, \ldots, N$, then the same axiom implies that $\tilde{p} \sim \tilde{p}'$, which leads us to a contradiction.

Once we have determined z, $z' \in Z$ and a in the support of p_A such that $[(z,a)] \succ [(z',a)]$, it also follows from **opportunity-comparable independence**—assuming, without loss of generality, $a = a_1$—that

$$\begin{aligned}([z],p_A) \;=\; & [(z,a_1),\lambda_1;(z,a_2),\lambda_2;\;...\;;(z,a_N),\lambda_N] \succ \\ & [(z',a_1),\lambda_1;(z,a_2),\lambda_2;\;...\;;(z,a_N),\lambda_N] = p'\end{aligned}$$

That is, there exists $([z],\ p_A)$, $p' \in \Delta(p_A)$ such that $([z],\ p_A) \succ p'$. It then follows that there exists $q' \in \Delta(q_A)$ such that $q' \succ p'$. To see this, note that if $([z],\ q_A) \succcurlyeq ([z],\ p_A)$, this conclusion is immediate. On the other hand, if $([z],\ p_A) \succ ([z],\ q_A)$, then by **importance of own outcomes**, there exists $\overline{z} \in Z$ such that $([\overline{z}],\ q_A) \succcurlyeq ([z],\ p_A)$. In this case too, the conclusion follows that there exists $q' \in \Delta(q_A)$ such that $q' \succ p'$. Once we have identified such p' and q', it is immediate that either condition (A) or (B) listed above holds. To see this, if $q' \succ p$, then (A) holds and if $q \succ p'$, then (B) holds. On the other hand, if neither hold, that is, $p \succcurlyeq p'$, $q' \succcurlyeq q$, then once again one or both of (A) and (B) must hold. □

We next prove a representation result for $\succcurlyeq$, which constitutes an intermediate step in proving the representation specified in Theorem 1.

Lemma 2. *There exists a function $W\colon \Delta \to \mathbb{R}$ that represents $\succcurlyeq$, with the property that for any p, $q \in \Delta$ with $p_A = q_A$,*

$$W(\lambda p + (1-\lambda)q) = \lambda W(p) + (1-\lambda)W(q).$$

Further, the function W is unique up to positive affine transformation.

Proof. Consider all subsets $\Delta(p_A)$, $p_A \in \Delta(A)$, of the set Δ. It is straightforward to verify that our three axioms of **weak order**, **Archimedean** and **opportunity-comparable independence** imply that $\succcurlyeq$ restricted to any such set $\Delta(p_A)$ satisfies the axioms of the vonNeumann-Morgenstern set up. Therefore, for any such set, there exists a function W_{p_A} : $\Delta(p_A) \to \mathbb{R}$ such that:

1. W_{p_A} represents $\succcurlyeq$ restricted to $\Delta(p_A)$, that is, for any q, $q' \in \Delta(p_A)$, $q \succcurlyeq q'$ iff $W_{p_A}(q) \geq W_{p_A}(q')$, and
2. For any q, $q' \in \Delta(p_A)$, $W_{p_A}(\lambda q + (1-\lambda)q') = \lambda W_{p_A}(q) + (1-\lambda)W_{p_A}(q')$.

Furthermore, any such function W_{p_A} is unique up to positive affine transformation.

We will now piece together the various such W_{p_A} functions to define the desired function W as in the statement of the lemma. Note that since we are considering the case where $\succ\ \neq \emptyset$, there exists p, $q \in \Delta$ such that $p \succ q$. By lemma 1 it follows that either $\Delta^*(p_A) \neq \emptyset$

or $\Delta^*(q_A) \neq \emptyset$ (or both)—without loss of generality, assume that it is the former. Begin by defining the function W on the set $\Delta(p_A)$ by setting $W(p') = W_{p_A}(p')$, for all $p' \in \Delta(p_A)$. Now, consider $q' \in \Delta$ such that $p' \succcurlyeq q' \succcurlyeq p''$ for some p', $p'' \in \Delta(p_A)$. By the **Archimedean** axiom and **opportunity-comparable independence**, it follows that there exists $q^* \in \Delta(p_A)$ such that $q' \sim q^*$. For such $q' \in \Delta$, set $W(q') = W(q^*)$.

Next consider $q' \in \Delta$ such that $q' \succ p'$ for all $p' \in \Delta(p_A)$. It follows from lemma 1 that $\Delta^*(p_A) \cap \Delta^*(q'_A) \neq \emptyset$. Now pick $\widetilde{p}$, $\widetilde{p}' \in \Delta(p_A)$, with $\widetilde{p} \succ \widetilde{p}'$, and $\widetilde{q}$, $\widetilde{q}' \in \Delta(q'_A)$ such that $\widetilde{p} \sim \widetilde{q}$ and $\widetilde{p}' \sim \widetilde{q}'$. Recall that the function $W_{q'_A}$ is unique up to positive affine transformation, that is, we have two degrees of freedom in specifying it. Accordingly, we can *redefine* it by setting $W_{q'_A}(\widetilde{q}) = W(\widetilde{p})$ and $W_{q'_A}(\widetilde{q}') = W(\widetilde{p}')$. We can then extend the function W to the set $\Delta(q'_A)$ by setting $W(\widehat{q}) = W_{q'_A}(\widehat{q})$ for all $\widehat{q} \in \Delta(q'_A)$. Note that **opportunity-comparable independence** guarantees that all our definitions made thus far are consistent.

We can similarly extend the function W to any $q' \in \Delta$ such that $p' \succ q'$ for all $p' \in \Delta(p_A)$. It is straightforward to verify that the function W, thus defined, satisfies the condition specified in the statement of the lemma. □

The following corollary follows: Any lottery $p \in \Delta$, for which p_A is degenerate, has an "expected utility" representation. However, it is important to recognize that this need not be true, in general, when p_A is non-degenerate.

Corollary 1. *For any $p \in \Delta$ such that $p_A = [a]$ for some $a \in A$,*

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a),$$

where the function $w : Z\times A \to \mathbb{R}$ is defined by setting $w(z,a) = W([(z,a)])$ for all $(z,a) \in Z\times A$.

We next define binary relations that will be useful in the further development of the proof. Consider p, p', q, $q' \in \Delta$ such that $p_A = p'_A$ and $q_A = q'_A$. We define the binary relation $\succcurlyeq^*$ (resp. $\succ^*$, $\sim^*$) on $\Delta \times \Delta$ as follows:

$$(p,p') \succcurlyeq^* \text{ (resp. } \succ^*, \sim^*) \ (q,q') \text{ if } .5p + .5q' \succcurlyeq \text{ (resp. } \succ, \sim) \ .5p' + .5q.$$

Note that $(p,p') \succ^* (q,q')$ iff $(p,p') \succcurlyeq^* (q,q')$ and $\neg[(q,q') \succcurlyeq^* (p,p')]$ and $(p,p') \sim^* (q,q')$ iff $(p,p') \succcurlyeq^* (q,q')$ and $(q,q') \succcurlyeq^* (p,p')$. The interpretation of these binary relations is straightforward. As discussed in the text, since $.5p + .5q'$ and $.5p' + .5q$ provide the others with the same overall opportunities, as do p and p', on one hand, and q and q', on the other, if DM prefers the lottery $.5p + .5q'$ to $.5p' + .5q$, it can be inferred that she considers the difference between p and p' to be at least as great as the difference between q and q'. Observe that, we may use the binary relation $\succcurlyeq^*$ to re-state **consistent evaluation of opportunities** as follows:

Consistent Evaluation of Opportunities: *Let* $\widetilde{p}$, $\widetilde{p}'$, $\widetilde{q}$, $\widetilde{q}' \in \Delta$ *be such that* $\widetilde{p}_A = \widetilde{p}'_A$, $\widetilde{q}_A = \widetilde{q}'_A$ *and* $\widetilde{p} \sim \widetilde{q}$, $\widetilde{p}' \sim \widetilde{q}'$. *If* q, $q' \in \Delta$ *with* $q_A = q'_A = \widetilde{q}_A$ *are such that* $(q, q') \sim^* (\widetilde{q}, \widetilde{q}')$, *then for any* p, $p' \in \Delta$ *with* $p_A = p'_A = \widetilde{p}_A$,

$$(p, p') \sim^* (q, q') \text{ if and only if } (p, p') \sim^* (\widetilde{p}, \widetilde{p}').$$

We will now develop a series of lemmas that will establish the connection between the function W defined in lemma 2 and the binary relation $\succcurlyeq^*$—we will show that $(p, p') \succcurlyeq^* (q, q')$ iff $W(p) - W(p') \geq W(q) - W(q')$. We proceed in that direction by proving the following lemma which establishes some basic properties that $\succcurlyeq^*$ inherits from the axioms on $\succcurlyeq$.

Lemma 3. *Let* p, p', q, q', r, $r' \in \Delta$ *be such that* $p_A = p'_A$, $q_A = q'_A$ *and* $r_A = r'_A$.

1. $(p, p') \succcurlyeq^* (q, q') \Leftrightarrow (p, \lambda p + (1 - \lambda)p') \succcurlyeq^* (q, \lambda q + (1 - \lambda)q')$, *for all* $\lambda \in [0, 1]$.
2. $(p, p') \succcurlyeq^* (q, q') \Leftrightarrow (\lambda p + (1 - \lambda)r, \lambda p' + (1 - \lambda)r') \succcurlyeq^* (\lambda q + (1 - \lambda)r, \lambda q' + (1 - \lambda)r')$, *for all* $\lambda \in [0, 1]$.

Proof. To establish the first conclusion, observe that:

$$\begin{aligned} .5p + .5(\lambda q + (1 - \lambda)q') &= \lambda(.5p + .5q) + (1 - \lambda)(.5p + .5q') \\ .5(\lambda p + (1 - \lambda)p') + .5q &= \lambda(.5p + .5q) + (1 - \lambda)(.5p' + .5q) \end{aligned}$$

Note that $(.5p + .5q')_A = (.5p' + .5q)_A$. Hence by **opportunity-comparable independence**, it follows that $.5p + .5q' \succcurlyeq .5p' + .5q$ if and only if

$$\lambda(.5p + .5q) + (1 - \lambda)(.5p + .5q') \succcurlyeq \lambda(.5p + .5q) + (1 - \lambda)(.5p' + .5q), \ \forall \ \lambda \in [0, 1].$$

That is,

$$(p, p') \succcurlyeq^* (q, q') \Leftrightarrow (p, \lambda p + (1 - \lambda)p') \succcurlyeq^* (q, \lambda q + (1 - \lambda)q'), \ \forall \ \lambda \in [0, 1].$$

The second conclusion is also an immediate implication of **opportunity-comparable independence**. To establish it, observe that

$$\begin{aligned} .5(\lambda p + (1 - \lambda)r) + .5(\lambda q' + (1 - \lambda)r') &= \lambda(.5p + .5q') + (1 - \lambda)(.5r + .5r') \\ .5(\lambda p' + (1 - \lambda)r') + .5(\lambda q + (1 - \lambda)r) &= \lambda(.5p' + .5q) + (1 - \lambda)(.5r + .5r') \end{aligned}$$

Once again, since $(.5p + .5q')_A = (.5p' + .5q)_A$, **opportunity-comparable independence** implies that $.5p + .5q' \succcurlyeq .5p' + .5q$ if and only if

$$\lambda(.5p + .5q') + (1 - \lambda)(.5r + .5r') \succcurlyeq \lambda(.5p' + .5q) + (1 - \lambda)(.5r + .5r'), \ \forall \ \lambda \in [0, 1].$$

That is,

$$(p,p')\succcurlyeq^*(q,q') \Leftrightarrow (\lambda p{+}(1-\lambda)r,\ \lambda p'{+}(1-\lambda)r')\succcurlyeq^*(\lambda q{+}(1-\lambda)r,\ \lambda q'{+}(1-\lambda)r'),\ \forall\lambda{\in}[0,1].$$

□

Lemma 4. *Suppose p, p', q, $q' \in \Delta$ are such that $p_A = p'_A = q_A = q'_A$. Then*

$$(p,p') \succcurlyeq^* (q,q') \Leftrightarrow W(p) - W(p') \geq W(q) - W(q').$$

Proof. $(p,p') \succcurlyeq^* (q,q') \Leftrightarrow .5p + .5q' \succcurlyeq .5p' + .5q \Leftrightarrow$ (by lemma 2) $.5W(p) + .5W(q') \geq .5W(p') + .5W(q) \Leftrightarrow W(p) - W(p') \geq W(q) - W(q')$. □

Lemma 5. *Suppose p, p', q, $q' \in \Delta$ are such that $p_A = p'_A$ and $q_A = q'_A$. Further, there exists $\widetilde{p}$, $\widetilde{p}' \in \Delta$, such that $\widetilde{p}_A = \widetilde{p}'_A = p_A$ and $\widetilde{p} \sim q$, $\widetilde{p}' \sim q'$. Then,*

$$(p,p') \succcurlyeq^* (q,q') \Leftrightarrow W(p) - W(p') \geq W(q) - W(q').$$

Proof. Observe that:

$$\begin{aligned} .5p + .5q' \succcurlyeq .5p' + .5q \quad &\Leftrightarrow \quad .5p + .5\widetilde{p}' \succcurlyeq .5p' + .5\widetilde{p} \\ &\qquad \text{(by } \textbf{consistent evaluation of opportunities}\text{)} \\ &\Leftrightarrow \quad .5W(p) + .5W(\widetilde{p}') \geq .5W(p') + .5W(\widetilde{p}) \text{ (by lemma 2)} \\ &\Leftrightarrow \quad W(p) - W(p') \geq W(\widetilde{p}) - W(\widetilde{p}') \end{aligned}$$

Further, since $\widetilde{p} \sim q$ and $\widetilde{p}' \sim q'$, the representation provided in lemma 2 implies that $W(\widetilde{p}) - W(\widetilde{p}') = W(q) - W(q')$. Accordingly it follows that:

$$(p,p') \succcurlyeq^* (q,q') \Leftrightarrow .5p + .5q' \succcurlyeq .5p' + .5q \Leftrightarrow W(p) - W(p') \geq W(q) - W(q').$$

□

Lemma 6. *Suppose p, p', q, $q' \in \Delta$ are such that $p_A = p'_A$ and $q_A = q'_A$. Then,*

$$(p,p') \succcurlyeq^* (q,q') \Leftrightarrow W(p) - W(p') \geq W(q) - W(q').$$

Proof. Observe that if either of the following conditions hold, then the result follows by lemma 5.

1. There exists $\hat{p}$, $\hat{p}' \in \Delta(p_A)$ such that $\hat{p} \sim q$, $\hat{p}' \sim q'$
2. There exists $\hat{q}$, $\hat{q}' \in \Delta(q_A)$ such that $\hat{q} \sim p$, $\hat{q}' \sim p'$

Therefore, assume that neither of these cases apply. By lemma 1, it follows that $\Delta^*(p_A) \cap \Delta^*(q_A) \neq \emptyset$. Accordingly, we can find $\widetilde{p}$, $\widetilde{p}'$, $\widetilde{q}$, $\widetilde{q}' \in \Delta^*(p_A) \cap \Delta^*(q_A)$, with $\widetilde{p}_A = \widetilde{p}'_A = p_A$ and $\widetilde{q}_A = \widetilde{q}'_A = q_A$, and $\lambda \in [0,1]$ such that:

$$\widetilde{p} \sim \widetilde{q},\ \widetilde{p}' \sim \widetilde{q}' \text{ and } (\widetilde{q},\widetilde{q}') \sim^* (q,\lambda q + (1-\lambda)q')$$

Observe that:

$$\begin{aligned}(\widetilde{q},\widetilde{q}') \sim^* (q,\lambda q + (1-\lambda)q') &\Leftrightarrow W(\widetilde{q}) - W(\widetilde{q}') = W(q) - W(\lambda q + (1-\lambda)q') \text{ (by lemma 4)}\\ &\Leftrightarrow W(\widetilde{q}) - W(\widetilde{q}') = W(q) - [\lambda W(q) + (1-\lambda)W(q')]\\ &\Leftrightarrow W(\widetilde{q}) - W(\widetilde{q}') = (1-\lambda)(W(q) - W(q'))\end{aligned}$$

Accordingly, finding the required $\widetilde{q}$, $\widetilde{q}'$ and λ involves choosing them such that $W(\widetilde{q}) - W(\widetilde{q}') = (1-\lambda)(W(q) - W(q'))$, which we can always do given $\Delta^*(p_A) \cap \Delta^*(q_A) \neq \emptyset$. Therefore,

$$\begin{aligned}(p,p') \succcurlyeq^* (q,q') &\Leftrightarrow (p,\lambda p + (1-\lambda)p') \succcurlyeq^* (q,\lambda q + (1-\lambda)q') \text{ (by lemma 3.1)}\\ &\Leftrightarrow (p,\lambda p + (1-\lambda)p') \succcurlyeq^* (\widetilde{p},\widetilde{p}')\\ &\qquad \text{(by \textbf{consistent evaluation of opportunities})}\\ &\Leftrightarrow W(p) - W(\lambda p + (1-\lambda)p') \geq W(\widetilde{p}) - W(\widetilde{p}') \text{ (by lemma 4)}\\ &\Leftrightarrow W(p) - [\lambda W(p) + (1-\lambda)W(p')] \geq W(\widetilde{p}) - W(\widetilde{p}') \text{ (by lemma 2)}\\ &\Leftrightarrow (1-\lambda)(W(p) - W(p')) \geq W(\widetilde{q}) - W(\widetilde{q}') \text{ (since, } \widetilde{p} \sim \widetilde{q} \text{ and } \widetilde{p}' \sim \widetilde{q}')\\ &\Leftrightarrow (1-\lambda)(W(p) - W(p')) \geq (1-\lambda)(W(q) - W(q'))\\ &\Leftrightarrow W(p) - W(p') \geq W(q) - W(q')\end{aligned}$$

□

Lemma 7. *Let p, p', q, $q' \in \Delta$ be such that $p_A = p'_A$ and $q_A = q'_A$. Then for any $\lambda \in [0,1]$,*

$$W(\lambda p + (1-\lambda)q) - W(\lambda p' + (1-\lambda)q') = \lambda W(p) + (1-\lambda)W(q) - [\lambda W(p') + (1-\lambda)W(q')].$$

Proof. Without loss of generality, assume that $|W(p) - W(p')| \geq |W(q) - W(q')|$. Then we can pick $\widetilde{p}$, $\widetilde{p}' \in \Delta(p_A)$ such that $W(\widetilde{p}) - W(\widetilde{p}') = W(q) - W(q')$. Accordingly, by lemma 6, $(q,q') \sim^* (\widetilde{p},\widetilde{p}')$. It then follows from lemma 3.2 that for any $\lambda \in [0,1]$:

$$(\lambda p + (1-\lambda)q,\ \lambda p' + (1-\lambda)q') \sim^* (\lambda p + (1-\lambda)\widetilde{p},\ \lambda p' + (1-\lambda)\widetilde{p}').$$

Lemma 6 then implies that:

$$W(\lambda p + (1-\lambda)q) - W(\lambda p' + (1-\lambda)q') = W(\lambda p + (1-\lambda)\widetilde{p}) - W(\lambda p' + (1-\lambda)\widetilde{p}').$$

Since, $\widetilde{p}_A = p_A = \widetilde{p}'_A = p'_A$, lemma 2, in turn, implies that:

$$W(\lambda p + (1-\lambda)q) - W(\lambda p' + (1-\lambda)q') = \lambda W(p) + (1-\lambda)W(\widetilde{p}) - [\lambda W(p') + (1-\lambda)W(\widetilde{p}')].$$

Finally, since $W(\widetilde{p}) - W(\widetilde{p}') = W(q) - W(q')$, we can conclude that:

$$W(\lambda p + (1-\lambda)q) - W(\lambda p' + (1-\lambda)q') = \lambda W(p) + (1-\lambda)W(q) - [\lambda W(p') + (1-\lambda)W(q')].$$

□

Observe that repeated use of the above lemma establishes the following result:

Corollary 2. Let p^n, $q^n \in \Delta$ be such that $p^n_A = q^n_A$, $n = 1, \ldots, N$. Then for $\lambda^n \in [0,1]$, $n = 1, \ldots, N$, with $\sum_{n=1}^{N} \lambda^n = 1$,

$$W(\sum_{n=1}^{N} \lambda^n p^n) - W(\sum_{n=1}^{N} \lambda^n q^n) = \sum_{n=1}^{N} \lambda^n W(p^n) - \sum_{n=1}^{N} \lambda^n W(q^n)$$

We can now complete the proof for the sufficiency of the axioms. For that define the function $V : \Delta \to R$ by

$$V(p) = \sum_{(z,a) \in Z \times A} p(z,a) w(z,a),$$

where $w : Z \times A \to \mathbb{R}$ is defined by setting $w(z,a) = W([(z,a)])$ for all $(z,a) \in Z \times A$. The following result then follows from using the property established in lemma 7.

Lemma 8. *For any p, $q \in \Delta$ such that $p_A = q_A$,*

$$W(p) - W(q) = V(p) - V(q).$$

Proof. Consider p, $q \in \Delta$ with $p_A = q_A$. We can write p and q as follows:

$$p = \sum_{n=1}^{N} \lambda_n . (p_{Z,a_n}, [a_n]),\ q = \sum_{n=1}^{N} \lambda_n . (q_{Z,a_n}, [a_n]),\ \text{where}$$

1. $p_A = q_A = [a_1, \lambda_1; \ldots, a_N, \lambda_N]$
2. p_{Z,a_n} (respectively, q_{Z,a_n}) $\in \Delta(Z)$, $n = 1, \ldots N$, is the conditional measure of p (respectively, q) on Z when others get the outcome a_n,
3. $(p_{Z,a_n}, [a_n])$ (respectively, $[q_{Z,a_n}, a_n]$) $\in \Delta$, $n = 1, \ldots N$, is the allocation-lottery that gives the others a_n for sure and DM's outcomes are determined by the lottery p_{Z,a_n} (respectively, q_{Z,a_n}).

Then by corollary 2 it follows that:

$$W(p) - W(q) = \sum_{n=1}^{N} \lambda_n W((p_{Z,a_n}, [a_n])) - \sum_{n=1}^{N} \lambda_n W((q_{Z,a_n}, [a_n]))$$

As noted in corollary 1,

$$W((p_{Z,a_n}, [a_n])) = V((p_{Z,a_n}, [a_n])) \text{ and } W((q_{Z,a_n}, [a_n])) = V((q_{Z,a_n}, [a_n])),\ n = 1, \ldots, N.$$

This, therefore, implies that:

$$W(p) - W(q) = \sum_{n=1}^{N} \lambda_n V((p_{Z,a_n}, [a_n])) - \sum_{n=1}^{N} \lambda_n V((q_{Z,a_n}, [a_n]))$$

Since V is an "expected utility functional" and, accordingly, linear in probabilities, it follows that:

$$W(p) - W(q) = V(\sum_{n=1}^{N}\lambda_n.(p_{Z,a_n},[a_n])) - V(\sum_{n=1}^{N}\lambda_n.(q_{Z,a_n},[a_n])).$$

That is,

$$W(p) - W(q) = V(p) - V(q).$$

$\square$

It therefore follows that for any $p_A \in \Delta(A)$, the function V, restricted to $\Delta(p_A)$, represents $\succcurlyeq$. Further the function V is linear in probabilities, that is, for any q, $q' \in \Delta(p_A)$, $V(\lambda q + (1-\lambda)q') = \lambda V(q) + (1-\lambda)V(q')$. Since the function W restricted to $\Delta(p_A)$ is also linear in probabilities, it follows that there exists constants $f(p_A) > 0$, $g(p_A)$ such that for any $q \in \Delta(p_A)$

$$W(q) = f(p_A)V(q) + g(p_A).$$

Finally, note that lemma 8 implies that $f(p_A) = 1$, which gives us our representation.

It is straightforward to verify the necessity of the axioms. We omit the details here. We next proceed to prove the (essential) uniqueness of the w and g functions. Suppose ($\widetilde{w} : Z \times A \to \mathbb{R}$, $\widetilde{g} : \Delta(A) \to \mathbb{R}$) is another AEU representation of $\succcurlyeq$ along with ($w : Z \times A \to \mathbb{R}$, $g : \Delta(A) \to \mathbb{R}$). We need to establish that there exists constants $\alpha > 0$, β such that:

$$\widetilde{w} = \alpha w + \beta \text{ and } \widetilde{g} = \alpha g.$$

The proof for the case of $\succ = \emptyset$ is straightforward. Consider any (z,a), $(z',a') \in Z{\times}A$. Since, $[(z,a)] \sim [(z',a')]$, it follows that $\widetilde{w}(z,a) = \widetilde{w}(z',a')$, i.e., $\widetilde{w}$ is a constant function. Let $\widetilde{w}(z,a) = \widetilde{w}^*$ for all $(z,a) \in Z{\times}A$. Similarly, $w(z,a)$ is also a constant function. Let $w(z,a) = w^*$ for all $(z,a) \in Z{\times}A$. Accordingly, $\widetilde{w}(z,a) = w(z,a) + \widetilde{w}^* - w^*$. Now consider any $p \in \Delta$. Since $p \sim [(z',a')]$, it follows that $W(p) = W([(z',a')])$. That is,

$$\sum_{(z,a)} p(z,a)w(z,a) + g(p_A) = w(z',a')$$

Accordingly, we have that: $w^* + g(p_A) = w^*$, or that, $g(p_A) = 0$ for all $p_A \in \Delta(A)$. A similar argument establishes that $\widetilde{g}(p_A)$ is also equal to 0 for all $p_A \in \Delta(A)$. So, for the case $\succcurlyeq = \emptyset$, the desired conclusion follows with $\alpha = 1$ and $\beta = \widetilde{w}^* - w^*$.

Next, we consider the case when $\succ \neq \emptyset$. The functions $W : \Delta \to \mathbb{R}$ and $\widetilde{W} : \Delta \to \mathbb{R}$ given by:

$$W(p) = \sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A)$$

$$\widetilde{W}(p) = \sum_{(z,a)\in Z\times A} p(z,a)\widetilde{w}(z,a) + \widetilde{g}(p_A)$$

represents $\succcurlyeq$ and have the property that for any $p_A \in \Delta(A)$ and q, $q' \in \Delta(p_A)$,

$$W(\lambda q + (1-\lambda)q') = \lambda W(q) + (1-\lambda)W(q') \text{ and } \widetilde{W}(\lambda q + (1-\lambda)q') = \lambda\widetilde{W}(q) + (1-\lambda)\widetilde{W}(q')$$

We know from lemma 1 that there exists $p_A \in \Delta(A)$ such that $\Delta^*(p_A) \neq \emptyset$. It follows that there exists constants $\alpha > 0$, β such that for any $q \in \Delta(p_A)$, $\widetilde{W}(q) = \alpha W(q) + \beta$. Now consider any $q \notin \Delta(p_A)$. If q such that that there exists p', $p'' \in \Delta(p_A)$ with $p' \succcurlyeq q \succcurlyeq p''$, then there exists $p^* \in \Delta(p_A)$ satisfying $p^* \sim q$. Accordingly, $\widetilde{W}(q) = \widetilde{W}(p^*)$ and $W(q) = W(p^*)$. Hence,

$$\widetilde{W}(q) = \widetilde{W}(p^*) = \alpha W(p^*) + \beta = \alpha W(q) + \beta.$$

Next consider $q \in \Delta$ such that $q \succ p'$ for all $p' \in \Delta(p_A)$. By lemma 1, we know that in this case $\Delta^*(q_A) \cap \Delta^*(p_A) \neq \emptyset$. In particular, since $\Delta^*(q_A) \neq \emptyset$, there exists constants $\alpha(q_A) > 0$, $\beta(q_A)$ such that for any $q' \in \Delta(q_A)$, $\widetilde{W}(q') = \alpha(q_A)W(q') + \beta(q_A)$. Further, there exists $\widetilde{p}, \widetilde{p}' \in \Delta(p_A)$, $\widetilde{q}, \widetilde{q}' \in \Delta(q_A)$ such that $\widetilde{p} \sim \widetilde{q} \succ \widetilde{p}' \sim \widetilde{q}'$. Accordingly, it follows that:

$$\widetilde{W}(\widetilde{q}) - \widetilde{W}(\widetilde{q}') = \widetilde{W}(\widetilde{p}) - \widetilde{W}(\widetilde{p}').$$

That is,

$$\alpha(q_A)[W(\widetilde{q}) - W(\widetilde{q}')] = \alpha[W(\widetilde{p}) - W(\widetilde{p}')]$$

Given that $W(\widetilde{q}) - W(\widetilde{q}') = W(\widetilde{p}) - W(\widetilde{p}') > 0$, it follows that $\alpha(q_A) = \alpha$. Further, since $\widetilde{W}(\widetilde{q}) = \widetilde{W}(\widetilde{p})$, we have

$$\alpha W(\widetilde{q}) + \beta(q_A) = \alpha W(\widetilde{p}) + \beta.$$

At the same time, $W(\widetilde{q}) = W(\widetilde{p})$ and accordingly $\beta(q_A) = \beta$. We can therefore conclude that:

$$\widetilde{w}(z,a) = \widetilde{W}([(z,a)]) = \alpha W([(z,a)]) + \beta = \alpha w(z,a) + \beta, \text{ for all } (z,a) \in Z \times A.$$

Now consider any $p \in \Delta$. Our representation implies that

$$\widetilde{g}(p_A) = \widetilde{W}(p) - \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\widetilde{w}(z,a).$$

That is,

$$\widetilde{g}(p_A) = \alpha W(p) + \beta - \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{\alpha w(z,a) + \beta\}.$$

That is,

$$\widetilde{g}(p_A) = \alpha[W(p) - \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a)].$$

Hence,

$$\widetilde{g}(p_A) = \alpha g(p_A).$$

7.2 Proof of Theorem 2

The proof of Theorem 1 establishes that if $\succcurlyeq$ satisfies **weak order**, **Archimedean**, **opportunity-comparable independence**, **consistent evaluation of opportunities** and **importance of own outcomes**, then there exists an AEU representation ($w : Z \times A \to \mathbb{R}$, $g : \Delta(A) \to \mathbb{R}$) of $\succcurlyeq$. That is, the function $W{:}\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A),$$

represents $\succcurlyeq$. Pick one such representation and the associated W function.

The key to establishing the sufficiency of the suggested axioms for the independent risk preference AEU representation is the following lemma.

Lemma 9. *For any z, $z' \in Z$ and a, $a' \in A$,*

$$w(z,a) - w(z,a') = w(z',a) - w(z',a')$$

Proof. Consider (z,a), $(z',a') \in Z{\times}A$. There are two possible cases: either $\neg([(z,a)] \sim [(z',a')])$—without loss of generality assume that $[(z,a)] \succ [(z',a')]$—or $[(z,a)] \sim [(z',a')]$. We will show that the conclusion of the lemma holds in each of these cases.

- **Case 1**: $[(z,a)] \succ [(z',a')]$

First consider the possibility that $[(z,a')] \sim [(z,a)]$. That is, for $\lambda = 1$,

$$[(z,a')] \sim \lambda[(z,a')] + (1-\lambda)[(z',a')]$$

Since, $[(z,a)] \sim [(z,a')]$, $[(z',a')] \sim [(z',a')]$ and others' opportunities are identical under $[(z,a')]$ and $[(z',a')]$, **strong independent standards of equity** implies that for $\lambda = 1$:

$$[(z',a)] \sim \lambda[(z',a')] + (1-\lambda)[(z,a')].$$

That is, $[(z',a)] \sim [(z',a')]$. Hence, in this case it follows that: $w(z,a) - w(z,a') = w(z',a) - w(z'.a') = 0$.

Next consider the possibility that $[(z,a')] \succ [(z,a)]$. **Strong independent standards of equity**, then, implies that $[(z',a')] \succ [(z',a)]$. Further, by **importance of own outcomes**, there exists $\overline{z}$ such that $[(\overline{z},a)] \succcurlyeq [(z,a')]$. Accordingly, there exists $p \in \Delta$ with $p_A = [a]$ such that $p \sim [(z,a')]$. **Strong independent standards of equity**, in turn, implies that there exists a unique $\lambda \in (0,1)$ such that:

$$[(z,a)] \sim \lambda p + (1-\lambda)[(z',a)] \text{ and } [(z',a')] \sim \lambda[(z',a)] + (1-\lambda)p.$$

Accordingly,

$$W([(z,a)]) = \lambda W(p) + (1-\lambda)W([(z',a)]) \text{ and } W([(z',a')]) = \lambda W([(z',a)]) + (1-\lambda)W(p).$$

That is,

$$w(z,a) = \lambda w(z,a') + (1-\lambda)w(z',a) \text{ and } w(z',a') = \lambda w(z',a) + (1-\lambda)w(z,a').$$

That is,

$$w(z,a) - w(z',a) = \lambda[w(z,a') - w(z',a)] \text{ and } w(z,a') - w(z',a') = \lambda[w(z,a') - w(z',a)].$$

Accordingly,

$$w(z,a) - w(z',a) = w(z,a') - w(z',a'); \text{ or, } w(z,a) - w(z,a') = w(z',a) - w(z',a').$$

Next consider the possibility that $[(z,a)] \succ [(z,a')] \succcurlyeq [(z',a')]$. **Importance of own outcomes** allows us to establish that there exists $p \in \Delta$ with $p_A = [a']$ such that $p \sim [(z,a)]$. **Strong independent standards of equity**, then, implies that there exists a unique $\lambda \in [0,1]$ such that:

$$[(z,a')] \sim \lambda p + (1-\lambda)[(z',a')] \text{ and } [(z',a)] \sim \lambda[(z',a')] + (1-\lambda)p.$$

We can then show, along similar lines as above, that $w(z,a) - w(z,a') = w(z',a) - w(z',a')$.

Finally, consider the case when $[(z',a')] \succ [(z,a')]$. **Strong independent standards of equity** implies that $[(z',a)] \succ [(z,a)]$. **Importance of own outcomes** allows us to establish that there exists $p \in \Delta$ with $p_A = [a']$ such that $p \sim [(z',a)]$. **Strong independent standards of equity**, then, implies that there there exists a unique $\lambda \in [0,1]$ such that

$$[(z',a')] \sim \lambda p + (1-\lambda)[(z,a')] \text{ and } [(z,a)] \sim \lambda[(z,a')] + (1-\lambda)p,$$

and it can be established along similar lines as above that $w(z,a) - w(z,a') = w(z',a) - w(z',a')$.

- **Case 2**: $[(z,a)] \sim [(z',a')]$

The result can be established in this case using similar arguments as above. We do not provide the details here. □

Fix $z' \in Z$, $a' \in A$ and define the functions $u : Z \to \mathbb{R}$ and $v : A \to \mathbb{R}$ as follows:

$$u(z) = w(z,a') \text{ and } v(a) = w(z',a) - w(z',a')$$

By lemma 9, it follows that:

$$w(z,a) - w(z,a') = w(z',a) - w(z',a')$$

That is,

$$w(z,a) = w(z,a') + [w(z',a) - w(z',a')] = u(z) + v(a).$$

Hence,

$$W(p) = \sum_{z \in Z} p_Z(z)u(z) + \sum_{a \in A} p_A(A)v(a) + g(p_A),$$

represents $\succcurlyeq$. The necessity of **strong independent standards of equity** for the representation is straightforward and we omit the details here.

To establish the essential uniqueness result, suppose that $(\widetilde{u}, \widetilde{v}, \widetilde{g})$ is another independent risk preference AEU representation of $\succcurlyeq$. Denote the function $\widetilde{W} : \Delta \to \mathbb{R}$ by,

$$\widetilde{W}(p) = \sum_{z \in Z} p_Z(z)\widetilde{u}(z) + \sum_{a \in A} p_A(a)\widetilde{v}(a) + \widetilde{g}(p_A).$$

Fix $z' \in Z$ and $a' \in A$. Then for any $z \in Z$ and $a \in A$ it follows that:

$$\widetilde{W}([(z,a)]) - \widetilde{W}([(z',a)]) = \widetilde{u}(z) - \widetilde{u}(z') \text{ and } \widetilde{W}([(z',a)]) - \widetilde{W}([(z',a')]) = \widetilde{v}(a) - \widetilde{v}(a').$$

Similarly,

$$W([(z,a)]) - W([(z',a)]) = u(z) - u(z') \text{ and } W([(z',a)]) - W([(z',a')]) = v(a) - v(a').$$

By the essential uniqueness result of Theorem 1, we know that there exists constants $\alpha > 0$ and β such that $\widetilde{W} = \alpha W + \beta$ and $\widetilde{g} = \alpha g$. Accordingly,

$$\widetilde{u}(z) - \widetilde{u}(z') = \alpha[u(z) - u(z')] \text{ and } \widetilde{v}(a) - \widetilde{v}(a') = \alpha[v(a) - v(a')]$$

That is,

$$\widetilde{u}(z) = \alpha u(z) + \beta' \text{ and } \widetilde{v}(a) = \alpha v(a) + \beta'',$$

where $\beta' = \widetilde{u}(z') - \alpha u(z')$ and $\beta'' = \widetilde{v}(a') - \alpha v(a')$.

7.3 Proof of Theorem 3

The proof of Theorem 1 establishes that if $\succcurlyeq$ satisfies **weak order**, **Archimedean**, **opportunity comparable independence**, **consistent evaluation of opportunities** and **importance of own outcomes**, then there exists an AEU representation ($w : Z \times A \to \mathbb{R}$, $g : \Delta(A) \to \mathbb{R}$) of $\succcurlyeq$. That is, the function $W{:}\Delta \to \mathbb{R}$, given by

$$W(p) = \sum_{(z,a) \in Z \times A} p(z,a)w(z,a) + g(p_A),$$

represents $\succcurlyeq$. Pick one such representation and the associated W function. We now establish that, in addition, when $\succcurlyeq$ also satisfies **weak independent standards of equity**, then there exists a taste-equity separable AEU representation of $\succcurlyeq$. Recall that the definition of a taste-equity separable AEU representation pre-fixes a $z^* \in Z$ and $a^* \in A$. We refer to these outcomes in the proof below. In addition, note that **weak independent standards of equity** implies that for any $z \in Z$ and a, $a' \in A$, $(z,a) \succcurlyeq (z,a')$ iff $(z^*,a) \succcurlyeq (z^*,a')$. In order to show the sufficiency of the axioms for the representation, we have to consider two cases.

- **Case 1**: $(z^*, a) \sim (z^*, a')$ for all a, $a' \in A$.

In this case $w(z, .) : A \to \mathbb{R}$ is a constant function for any $z \in Z$. Define the functions $u : Z \to \mathbb{R}$, $\theta : Z \to \mathbb{R}_{++}$ and $v : A \to \mathbb{R}$ as follows:

$$u(z) = w(z, a^*),\ \theta(z) \equiv 1,\ v(a) = w(z^*, a).$$

Accordingly,

$$\begin{aligned} w(z, a) &= w(z, a^*) + w(z^*, a) - w(z^*, a^*) \\ &= u(z) + \theta(z)(v(a) - v(a^*)) \end{aligned}$$

- **Case 2**: $[(z^*, \overline{a})] \succ [(z^*, \underline{a})]$ for some $\overline{a}$, $\underline{a} \in A$

The key to establishing the representation in this case is the following lemma.

Lemma 10. *Suppose a, a', $a'' \in A$ are such that $[(z^*, a')] \succcurlyeq [(z^*, a)] \succcurlyeq [(z^*, a'')]$, with $[(z^*, a')] \succ [(z^*, a'')]$. Then, there exists a unique $\lambda \in [0, 1]$ such that for any $z \in Z$,*

$$w(z, a) = \lambda w(z, a') + (1 - \lambda)w(z, a'').$$

Proof. Note that since $[(z^*, a')] \succ [(z^*, a'')]$, by **importance of own outcomes**, it follows that there exists $\underline{z} \in Z$ such that $[(z^*, a'')] \succcurlyeq [(\underline{z}, a')]$. Accordingly, there exists $p' \in \Delta$ with $p'_A = [a']$ such that $p' \sim [(z^*, a'')]$. It follows, then, that there exists a unique $\lambda \in [0, 1]$ such that:

$$[(z^*, a)] \sim \lambda[(z^*, a')] + (1 - \lambda)p'$$

Next consider any other $z \in Z$. It follows from **weak independent standards of equity** that $[(z, a')] \succcurlyeq [(z, a)] \succcurlyeq [(z, a'')]$, with $[(z, a')] \succ [(z, a'')]$. By a similar argument as above, we can establish that there exists q, $q' \in \Delta$ with $q_A = q'_A$ such that $q \sim [(z, a')]$ and $q' \sim [(z, a'')]$. It then follows from **weak independent standards of equity** that:

$$[(z, a)] \sim \lambda q + (1 - \lambda)q'.$$

Accordingly, the result follows. □

Next note that there exists $a' \in A$ such that either $[(z^*, a')] \succ [(z^*, a^*)]$ or $[(z^*, a^*)] \succ [(z^*, a')]$. We consider the proof for the former case; the latter case can be handled along similar lines. Consider any $a \in A$. Suppose $[(z^*, a')] \succcurlyeq [(z^*, a)] \succcurlyeq [(z^*, a^*)]$. Then by the above lemma, it follows that there exists a unique $\lambda \in [0, 1]$ such that for z^* and any other $z \in Z$:

$$w(z^*, a) = \lambda w(z^*, a') + (1 - \lambda)w(z^*, a^*) \text{ and } w(z, a) = \lambda w(z, a') + (1 - \lambda)w(z, a^*).$$

That is,

$$\frac{w(z,a)-w(z,a^*)}{w(z,a')-w(z,a^*)} = \lambda = \frac{w(z^*,a)-w(z^*,a^*)}{w(z^*,a')-w(z^*,a^*)}$$

That is,

$$w(z,a) = w(z,a^*) + \frac{w(z,a')-w(z,a^*)}{w(z^*,a')-w(z^*,a^*)}(w(z^*,a) - w(z^*,a^*)).$$

Next consider any $a \in A$ such that $[(z^*, a)] \succ [(z^*, a')]$. In this case, there exists a unique $\lambda \in [0, 1]$ such that for z^* and any other $z \in Z$:

$$w(z^*, a') = \lambda w(z^*, a) + (1-\lambda)w(z^*, a^*) \text{ and } w(z, a') = \lambda w(z, a) + (1-\lambda)w(z, a^*).$$

That is,

$$\frac{w(z,a)-w(z,a^*)}{w(z,a')-w(z,a^*)} = \frac{1}{\lambda} = \frac{w(z^*,a)-w(z^*,a^*)}{w(z^*,a')-w(z^*,a^*)}$$

That is,

$$w(z,a) = w(z,a^*) + \frac{w(z,a')-w(z,a^*)}{w(z^*,a')-w(z^*,a^*)}(w(z^*,a) - w(z^*,a^*)).$$

It can be shown along similar lines that the above relationship also holds for the case when $[(z^*, a^*)] \succ [(z, a)]$. Now, define the functions $u : Z \to \mathbb{R}$, $\theta : Z \to \mathbb{R}_{++}$, $v : A \to \mathbb{R}$ as follows:

$$u(z) = w(z, a^*),\ \theta(z) = \frac{w(z,a')-w(z,a^*)}{w(z^*,a')-w(z^*,a^*)} \text{ and } v(a) = w(z^*, a).$$

Accordingly, it follows that for any $(z, a) \in Z \times A$,

$$w(z, a) = u(z) + \theta(z)(v(a) - v(a^*)).$$

Therefore, (u, θ, v, g) is a taste-equity separable AEU representation of $\succcurlyeq$. The necessity of **weak independent standards of equity** for the representation is straightforward and we omit the details here.

We next prove the essential uniqueness result. Suppose $(\widetilde{u}, \widetilde{\theta}, \widetilde{v}, \widetilde{g})$ is another taste-equity separable AEU representation. Observe that for any a, a', $a'' \in A$, with $v(a') > v(a'')$, and $\lambda \in [0, 1]$,

$$v(a) = \lambda v(a') + (1-\lambda)v(a'') \text{ if and only if } \widetilde{v}(a) = \lambda\widetilde{v}(a') + (1-\lambda)\widetilde{v}(a'').$$

It then follows that there exists constants $\alpha' > 0$, β' such that for any $a \in A$,

$$\widetilde{v}(a) = \alpha' v(a) + \beta'.$$

Next, denote the function $\widetilde{W} : \Delta \to \mathbb{R}$ by

$$\widetilde{W}(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)\{\widetilde{u}(z) + \widetilde{\theta}(z)(\widetilde{v}(a) - v(a^*))\} + \widetilde{g}(p_A).$$

From the proof of the essential uniqueness result of theorem 1, we know that that there exists constants α, β such that for any $p \in \Delta$, $\widetilde{W}(p) = \alpha W(p) + \beta$. Further, note that for any $z \in Z$ and a, a' with $v(a) > v(a')$, we have

$$\widetilde{W}([(z,a)]) - \widetilde{W}([(z,a')]) = \widetilde{\theta}(z)[\widetilde{v}(a) - \widetilde{v}(a')]$$

and

$$W([(z,a)]) - W([(z,a')]) = \theta(z)[v(a) - v(a')].$$

That is,

$$\widetilde{\theta}(z) = \tfrac{\widetilde{W}([(z,a)]) - \widetilde{W}([(z,a')])}{\widetilde{v}(a) - \widetilde{v}(a')} \text{ and } \theta(z) = \tfrac{W([(z,a)]) - W([(z,a')])}{v(a) - v(a')}.$$

It then follows that:

$$\widetilde{\theta}(z) = \tfrac{\alpha}{\alpha'}\theta(z).$$

But, since $\widetilde{\theta}(z^*) = \theta(z^*)$, it follows that $\alpha' = \alpha$ and, accordingly, $\widetilde{\theta}(z) = \theta(z)$ for all $z \in Z$.

Finally note that for any z, $z' \in Z$, $a \in A$,

$$\widetilde{u}(z) - \widetilde{u}(z') = \widetilde{W}([(z,a)]) - \widetilde{W}([(z',a)]) - (\widetilde{\theta}(z) - \widetilde{\theta}(z'))(\widetilde{v}(a) - \widetilde{v}(a^*)),$$

and

$$u(z) - u(z') = W([(z,a)]) - W([(z',a)]) - (\theta(z) - \theta(z'))(v(a) - v(a^*)).$$

Accordingly,

$$\widetilde{u}(z) - \widetilde{u}(z') = \alpha\{W([(z,a)]) - W([(z',a)])\} - (\theta(z) - \theta(z'))\alpha(v(a) - v(a^*)).$$

That is,

$$\widetilde{u}(z) - \widetilde{u}(z') = \alpha(u(z) - u(z')).$$

Accordingly,

$$\widetilde{u}(z) = \alpha u(z) + \beta.$$

7.4 Proof of Proposition 1

We will establish the result for an independent risk preference AEU representation. The result can be established along similar lines for a taste-equity separable AEU representation. So, consider an independent risk preference AEU representation (u:$Z\to\mathbb{R}$, v:$A\to\mathbb{R}$, g:$\Delta(A)\to\mathbb{R}$) of $\succcurlyeq$. That is, the function W:$\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{z\in Z} p_Z(z)u(z) + \sum_{a\in A} p_A(a)v(a) + g(p_A),$$

represents $\succcurlyeq$.

Suppose that, in addition, $\succcurlyeq$ satisfies **overweighing**. Consider any $p = ([z], p_A) = [(z, a^1), \lambda_1; \ . \ . \ . \ ; (z, a^N), \lambda_N] \in \Delta$ and let $(\underline{\lambda}, \underline{p}, \overline{p})$ be a bad-good decomposition of p. In particular, let $q^1, \ . \ . \ . \ , q^N \in \Delta$ be such that $q^1_A = \ . \ . \ . = q^N_A$, $[(z, a^n)] \sim q^n$ for all $n = 1, \ . \ . \ . \ , N$ and (z, a^n) is in the support of $\overline{p}$ iff $[(z, a^n)] \sim q^n \succcurlyeq \lambda_1 q^1 + \ . \ . \ . + \lambda_N q^N$. Further, let $\underline{p}^* = \sum_{\{n:(z,a^n)\in supp(\underline{p})\}}(\lambda_n/\underline{\lambda})q^n$ and $\overline{p}^* = \sum_{\{n:(z,a^n)\in supp(\overline{p})\}}(\lambda_n/1-\underline{\lambda})q^n$ be the bad and good outcomes equivalent of p. **Overweighing** implies that $p \sim \underline{\lambda}^2\underline{p}^* + (1-\underline{\lambda}^2)\overline{p}^*$. Since, $\underline{p}^*_A = \overline{p}^*_A$, it follows that,

$$W(p) = \underline{\lambda}^2 W(\underline{p}^*) + (1-\underline{\lambda}^2)W(\overline{p}^*)$$

Further, since $q^1_A = \ . \ . \ . = q^N_A$,

$$W(p) = \underline{\lambda}^2 . \sum_{\{n:(z,a^n)\in supp(\underline{p})\}} (\frac{\lambda_n}{\underline{\lambda}})W(q^n) + (1-\underline{\lambda}^2). \sum_{\{n:(z,a^n)\in supp(\overline{p})\}} (\frac{\lambda_n}{1-\underline{\lambda}})W(q^n)$$

That is,

$$W(p) = \underline{\lambda}^2 . \sum_{\{n:(z,a^n)\in supp(\underline{p})\}} (\frac{\lambda_n}{\underline{\lambda}})W([(z,a^n)]) + (1-\underline{\lambda}^2). \sum_{\{n:(z,a^n)\in supp(\overline{p})\}} (\frac{\lambda_n}{1-\underline{\lambda}})W([(z,a^n)])$$

That is,

$$u(z) + \sum_{n=1}^{N} \lambda_n v(a^n) + g(p_A) =$$

$$u(z) + \underline{\lambda}^2 . \sum_{\{n:(z,a^n)\in supp(\underline{p})\}} (\frac{\lambda_n}{\underline{\lambda}})v(a^n) + (1-\underline{\lambda}^2). \sum_{\{n:(z,a^n)\in supp(\overline{p})\}} (\frac{\lambda_n}{1-\underline{\lambda}})v(a^n)$$

That is,

$$\sum_{n=1}^{N} \lambda_n v(a^n) + g(p_A) = \underline{\lambda} . \sum_{\{n:(z,a^n)\in supp(\underline{p})\}} \lambda_n v(a^n) + (1+\underline{\lambda}). \sum_{\{n:(z,a^n)\in supp(\overline{p})\}} \lambda_n v(a^n)$$

That is,

$$g(p_A) = \underline{\lambda} \sum_{n=1}^{N} \lambda_n v(a^n) + \sum_{\{n:(z,a^n)\in supp(\overline{p})\}} \lambda_n v(a^n) - \sum_{n=1}^{N} \lambda_n v(a^n)$$

That is,

$$\begin{aligned} g(p_A) &= \underline{\lambda} \sum_{n=1}^{N} \lambda_n v(a^n) - \sum_{\{n:(z,a^n)\in supp(\underline{p})\}} \lambda_n v(a^n) \\ &= \sum_{n=1}^{N} \lambda_n . \max\{\sum_{\tilde{n}=1}^{N} \lambda^{\tilde{n}} v(a^{\tilde{n}}) - v(a^n), 0\} \end{aligned}$$

Hence, the sufficiency of **overweighing** for asymmetric adjustment follows. Establishing the necessity of the axiom is straightforward and we omit the details.

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