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Rich Vehicle Routing in Theory and Practice

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Rich Vehicle Routing in Theory and Practice

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Abstract

The contribution of this paper is a comparison of the state-of-the-art of academic research on and commercial software for modelling and solving vehicle routing problems. To this end, the paper presents a compact review of vehicle routing literature and an overview of the results of a recent study of commercial vehicle routing software systems with respect to the problem features these systems are able to handle and the solution methods the systems use for automatic generation of vehicle routes. In this way, existing application and research gaps are identified.
Keywords: Rich vehicle routing; Commercial vehicle routing software; Heuristics

1 Introduction

Vehicle routing is a central task in a large number of private and public corporations. Tours have to be planned in very diverse sectors of the economy, not only in the logistics and transport business, but in virtually all industrial sectors producing physical goods. In addition to transport on public roads, applications of vehicle routing can also be found in intra-plant logistics, that is, local transport within a factory or warehouse building or on company premises.
Beside the considerable importance of effective and efficient vehicle routing for the enterprises themselves, the macroeconomic relevance of vehicle routing must not be overlooked: The avoidance of unnecessary or unnecessarily long tours with low capacity utilization removes pressure from road infrastructure, improves traffic flow for freight as well as passenger transport, and, by reducing emissions, makes a sustained contribution to the protection of the environment.
For operational research, vehicle routing constitutes one of its great success stories. Vehicle routing problems (VRPs) in their many variants have been the subject of intensive study for more than half a century now. This has led to the publication of thousands of academic papers and to the foundation of numerous software companies worldwide selling commercial vehicle routing software. This development is certainly due to the intellectual challenge VRPs pose as well as to their practical relevance in logistics and transport. Research on VRPs is incessantly ongoing, stimulated by unsolved theoretical problems and continuous input from logistics practice.
The contribution of this paper is a comparison of the state-of-the-art of academic research on and commercial software for modelling and solving vehicle routing problems. To this end, the paper presents a compact review of vehicle routing literature and an overview of the results of a recent study of commercial vehicle routing software systems (CVRSs) with respect to the problem features these systems are able to handle and the solution methods the systems use for automatic generation of vehicle routes (Drexl [14]). In this way, existing application and

research gaps are identified. This should be of interest for VRP researchers, and also logistics practitioners using or planning to use CVRSs should benefit from this paper, by learning about the potential of modern CVRSs.

Throughout the paper, the following definitions apply. The fundamental activity to be planned in vehicle routing is called a *request*. A request may be a transport order, such as the delivery of a shipment from a central depot to a recipient, the pickup of a shipment from a consignor and the transfer to a central depot, the pickup of a shipment at some location and the transport to some other location, or a visit at a location to perform a service there, without picking up or delivering a physical good. *Vehicle routing* means to group requests into clusters performed by one vehicle each, and to determine, for each cluster, a complete sequence of the resulting locations to be visited. This process can be performed manually by a human planner, automatically by a computer program executing a mathematical algorithm, or by a combination of both. The goal of vehicle routing is the optimization of an objective function. This will regularly be the minimization of a cost function, of the number of used vehicles, of the total distance travelled etc.

The rest of the paper is structured as follows. The next section describes, in a nutshell, the state-of-the-art of academic VRP research. Section 3 presents the results of a comprehensive study of the German CVRS market, focussing on modelling and algorithmic aspects for the automatic solution of VRPs. Section 4 then discusses the gap between theory and practice, and Section 5 gives a conclusion and an outlook.

2 State-of-the-art of academic VRP research

As stated in the introduction, over the last half century, there have been thousands of academic publications on vehicle routing. Therefore, the following elaborations can of course only give a very rough overview (a 'survey of surveys') and must necessarily refer the reader to the literature for details.

The archetypal academic VRP, the *capacitated vehicle routing problem* (CVRP) can be described as follows. Given are a set of identical vehicles stationed at one depot and equipped with a limited loading capacity, and a set of geographically dispersed customers with a certain demand for a homogeneous good. The task is to determine an optimal (with respect to an objective function) tour plan, that is, a set of vehicle routes, specifying which customers are visited by which vehicle in which sequence, such that each customer is visited exactly once, the complete demand of each customer is satisfied, and the loading capacity of the vehicles is maintained on each tour. The objective is to minimize overall cost or travelled distance.

The existing VRP literature can be divided into theoretical papers studying models or methods for idealized or standardized problems and problem-oriented case studies dealing with concrete real-world applications. The former class considers exact as well as heuristic approaches and uses theoretical benchmark instances to measure the effectiveness of the devised algorithms. (A large number of benchmark instances for different types of VRP can be found at `http://people.brunel.ac.uk/~mastjjb/jeb/info.html`.) For the latter class, the term 'rich vehicle routing' has been coined rather recently to denote models and solution approaches for problems that feature several or all aspects of a real-world application. Most papers belonging to the latter class focus on one new or particularly interesting or difficult aspect. A number of important such aspects was queried in the CVRS study mentioned above and is discussed in detail in Section 3.3.

The most important fundamental variants of the CVRP are:

- VRPs with time windows (VRPTW)
  In the VRPTW, the service at each customer must start within a given time window.
- Split delivery VRPs (SDVRP)
  In the SDVRP, customers may be visited more than once by more than one vehicle. Each vehicle may deliver a fraction of a customer's demand.

- Pickup-and-delivery problems (PDP)
  In the CVRP and the VRPTW, all goods or shipments to be delivered are picked up at the central depot. In the PDP, the tasks consist in the transport of shipments from one location to another, that is, not only the delivery locations are all different, but also the pickup locations.
- Capacitated arc routing problems (CARP)
  The CARP is a variant of the CVRP where the task is not to visit customers to perform a service, but where the service is performed while travelling along the links of a (road) network.
- Location-routing problems (LRP)
  LRPs combine routing and locational decisions. The task is to determine a set of vehicle routes and the locations/facilities/depots where each route starts and ends.
- Stochastic VRPs
  In usual VRPs, all data is assumed to be deterministic. In stochastic VRPs, information on occurrence and volume of customer demand or travel times between customers is given by probability distributions.
- Dynamic VRPs
  In dynamic VRPs, the planner is forced to make decisions before all relevant information becomes available; decisions must then be modified as new information is received. Essentially, planning is performed parallel to plan execution.
- Inventory routing problems (IRP)
  In IRPs, there are no customer demands. Instead, each customer has a given consumption rate of a good, a given initial stock and a given storage capacity. The depot has to perform zero or more deliveries to each customer during a multi-period planning horizon to ensure that no customer runs out of stock. The objective is to plan routes of minimal cost for the deliveries.

With respect to models, VRPs are usually represented as graphs or networks and formulated as mixed-integer linear programs (MILPs). With respect to solution algorithms, two principal approaches can be distinguished. The first one is mathematical programming, the second one is heuristics and metaheuristics. Mathematical programming is based on MILP models of VRPs and, in theory, guarantees to find an optimal solution if one exists. However, mathematical programming algorithms can have prohibitive requirements with respect to computation time and memory requirements for larger instances. Moreover, the computation times for instances of the same size and structure may vary extremely. At the time of this writing, CVRP and VRPTW instances of more than 200 customers cannot be consistently solved to optimality. The most successful exact approaches are branch-and-cut-and-price methods, which combine cut and column generation with branch-and-bound.

Heuristics and metaheuristics are mostly based on graph-theoretic models. Heuristics and metaheuristics do not offer an optimality guarantee, but they overcome the limitations of exact algorithms and are able to find close-to-optimal solutions in short time, even for very large instances. Section 3.4 contains an extensive list of constructive and improvement heuristics as well as metaheuristics. Best-known heuristic solutions to benchmark instances have been computed with many different methods, so there is definitely no silver bullet. However, it must be noted that most successful heuristic approaches are so-called *hybrid* procedures combining several 'classical' ones.

Recent monographs on VRPs and their variants are Toth/Vigo [53] and Golden et al. [28]. Cordeau et al. [10] give an extensive survey of research on the CVRP, the VRPTW, stochastic VRPs and the IRP. Powell et al. [46] give an overview of dynamic VRPs, Nagy/Salhi [40] present a survey of location-routing problems, Archetti/Speranza [2] present a survey on split delivery VRPs, Parragh et al. [42], [43] present two surveys on the many variants of pickup-and-delivery problems, Laporte [36] reports on the last fifty years of academic vehicle routing from a historical perspective, and Corberán/Prins [8] give an extensive survey on arc routing problems. All of these references contain results on exact as well as heuristic methods.

Method-oriented surveys or tutorials are given by Funke et al. [20] (local search), Ahuja et al. [1] (very large-scale neighbourhood search), Powell [45] (adaptive dynamic programming), Glover/ Kochenberger [23], Cotta et al. [11] (metaheuristics), Desaulniers et al. [13] (MIP/column generation), Røpke [49] (MIP, large neighbourhood search), and Maniezzo et al. [39] (matheuristics). In addition, Gendreau/Potvin [21] give an integrating and unifying overview of metaheuristics, and Gendreau et al. [22] present a categorized bibliography of metaheuristics for several types of VRP. Finally, Baldacci et al. [3] describe an exact solution framework for different types of VRP which outperforms all other exact methods published so far and solves several previously unsolved benchmark instances.

Seminal case studies describing the successful solution of rich real-world VRPs include Xu et al. [55], Hartl et al. [30], Hollis et al. [31], Cheung et al. [7], Irnich [34], Zäpfel/Bögl [56], Ceselli et al. [6], Bock [4], Oppen et al. [41], Rieck/Zimmermann [48], and Schmid et al. [50].

3 CVRSs: A comprehensive study of the German market

A CVRS is a computer program which allows to

- Read in and display data on vehicle depots, customers, distances and travel times between locations, on requests, vehicles and drivers
- Construct, save and display vehicle routes
- Determine a complete route plan for a given data set (a *problem instance*) by calling an implementation of a mathematical algorithm, possibly after entering a set of parameters, without further user interaction.

The typical components of a CVRS are:

- An interface to a database or ERP system
- A GIS module for geocoding address data, computing distance and travel time matrices, and visualizing data and solutions in digital maps
- A planning module for automatic, manual, and interactive planning
- A telematics module for data exchange between vehicles and the dispatching office, and for tracking and tracing of vehicles (see Goel [24] for details)
- A statistics module for computing key performance indicators and reports

Usually, but not exclusively, CVRSs are used for planning tours of motor vehicles on public roads. In this case, a CVRS is often embedded into a *transport management system* (TMS). A TMS contains components for data entry, planning, administration, execution, control, and billing of transport services. TMSs and other software systems for logistics and transport are thoroughly discussed in Crainic et al. [12].

VRPs are, in essence, highly complex mathematical optimization problems. Because of this complexity, software for supporting human planners and decision makers has been widely used for years. There is a considerable number of manufacturers of CVRSs, and many of these manufacturers have been in the business for decades. Reasons for the use of CVRSs, as specified by users, are:

- Cost of execution and planning of tours is reduced, and efficiency is increased
- Dispatchers are relieved from routine jobs
- Possibilities for transport monitoring and surveillance as well as for statistics and controlling are improved
- Quality and transparency of planning is improved
- Dependency of the company on single persons and their knowledge is reduced
- Work of the sales department (of freight forwarders) is simplified, because faster and more precise pricing of ad-hoc customer requests becomes possible
- Work processes are unified and streamlined

The benefit of the practical use of CVRSs is substantiated by several scientific studies. See, for example, the literature survey in Eibl [17], p. 45 ff.

3.1 Structure of the study

For compilation of the study, a thorough search for CVRS manufacturers active on the German market was performed, and no less than 50 firms could be found. All of them were asked to fill in a detailed questionnaire containing more than 500 pieces of information on relevant aspects of the company and the CVRS in nine categories:

1. Company
2. Product
3. IT and software engineering
4. User interface
5. GIS component
6. Telematics
7. Models and algorithms for automatic vehicle routing
8. Reporting, KPI and statistics
9. Prices

The questions posed were, to a large extent, either of the multiple-choice or the yes-no type. The obtained information was evaluated on an aggregate level, by summing or averaging over all questionnaires. No information on single manufacturers or systems is given. The answers in the returned questionnaires were checked for plausibility; nevertheless, correctness could of course not be verified. However, the published results being aggregated, no vendor had anything to gain by exaggerating the capabilities of his product. With respect to content, aspects that were considered relevant by the study author, based on his own professional experience, were queried.

OR/MS Today, the journal edited by the Institute for Operations Research and the Management Sciences (INFORMS), features, in a two-year cycle, a survey on CVRSs for the North American market. The latest one, from February 2010, comprises 16 vendors (see `http://www.lionhrtpub.com/orms/orms-2-10/frsurvey.html`, `http://www.lionhrtpub.com/orms/surveys/Vehicle_Routing/vrss.html`). The results of both studies are hard to compare, because different information was gathered and organized differently. Only one company appeared in both surveys.

Sörensen et al. [51] refer to the 2006 version of the OR/MS survey. They list several characteristics of real-world VRPs, all of which are also contained in the study excerpted subsequently. They state (p. 241): 'Although there is an increasing academic focus on so-called "rich" vehicle routing problems (that incorporate more complex constraints and objectives), they have not in any way caught up with the whole complexity of real-life routing problems.' To a large extent, this point is also supported by the results presented in the present paper.

3.2 General results

28 companies sent back a filled-in questionnaire. This is a return rate of 56 %, which is acceptable.

3.2.1 Company structure and size

Most CVRS companies have their headquarters in Germany. The number of employees is 36 on average. The first manufacturer of 'software for logistics' was founded in 1961. The first CVRS, that is, vehicle routing software featuring an automatic, algorithm-based planning component, was offered in 1979.

All firms offer launching and rollout support as well as user-specific adaptation and customization of their software. (This shows that CVRSs are (still) not a standard, off-the-shelf product.) In addition, most companies use their own software for project work and consulting services.

Most firms, but not all, consider the algorithms used in their systems a core competence. Only four companies do not possess the source code of the algorithms and do not hold exclusive

rights on the code. These companies specialize in transport management systems (TMS) and use third-party components for automatic planning.
Cooperation with academia is common. More than three quarters of all firms stated that they cooperate with at least one university. This is mostly done in the form of master's and Ph.D. theses. 12 firms offer a free test or demo version, and several firms offer academic licences for use in teaching.
A basic, single-user licence for commercial use costs 15,000 Euros on average. This does not include customizing and preparatory training of users.

3.2.2 Industry sectors using CVRSs

As for the industry sectors using CVRSs and the respective number of sold licences, no reliable data could be gathered. Some firms provided detailed data, others did not or gave only aggregated information. However, a quintessential finding is that numerous CVRS firms have customers from the following sectors:

- Industry (raw materials and (semi-)finished goods transport)
- Wholesale and retail trade (consumer goods distribution)
- LTL and FTL forwarders
- Parcel delivery and letter mail services
- Reverse logistics and waste collection
- Service technician, salesman, and other staff dispatching
- Intralogistics

3.3 Models and algorithms for automatic vehicle routing in CVRSs

Obviously, a central part of a CVRS and the most interesting one from an OR perspective is the automatic planning component. Therefore, detailed information on this aspect was queried. This part of the questionnaire was answered by 27 participants.

3.3.1 General features of the automatic planning component

The following general features are supported by most systems:

- The solution of general pickup-and-delivery problems as described above with up to 10,000 requests
- The computation of a feasible solution within 5 minutes for arbitrary instances (when distance and travel time matrices are given)
- Multi-period planning horizons and multiple use of vehicles
- The possibility to determine only one part of the solution of a VRP, namely, to cluster the requests into groups which are to be performed by one vehicle (the other part, the sequencing of requests, is left to the dispatcher, or, even more often, to the drivers)
- The automatic assignment of vehicles and drivers to such groups
- A re-optimization component capable of computing a new feasible solution from an existing one after small changes to the instance (such as the arrival of some new requests; this can be considered a prerequisite for real-time planning)
- The possibility to limit the duration of an optimization run by specifying a maximal number of iterations and a maximal running time
- A batch mode for automatic execution of optimization runs with specific parameter settings

Interestingly, only 8 firms claim to be able to solve arc routing and postman problems. Moreover, although there are several papers on the topic (see the survey by Wäscher et al. [54]), only 6 systems contain a module for optimizing storage space utilization (using 3D packing algorithms).

3.3.2 Modelling features

3.3.2.1 Vehicle-related features

A homogeneous fleet is rarely found in the real world. Therefore, the ability to consider heterogeneous vehicles is an absolute must for any CVRS. Vehicles may differ with respect to several criteria:

- Cost:
  - Fixed
  - Variable:
    - Distance-dependent
      Distance-dependent costs may include road tolls.
    - Time-dependent
      Time-dependent costs may be linear or nonlinear and may include overtime pay or daily allowances for drivers.
    - Stop-dependent
  - Tariffs
    Tariffs used to be mandatory in Germany until the end of the 20th century and were dependent on goods types, weight, distance, time etc.; although the numeric values have decreased sharply, the calculation formulas are still often used in practice.
  - Penalty cost
    Penalty costs are often used to consider soft constraints or undesired but not strictly infeasible properties of solutions, or to allow infeasible solutions during the solution process.
- Capacity
  The most common capacity constraints in goods transport are:
  - Weight/payload
  - Volume
  - Loading metres
  - Number of pallet places

  Several of the above may be relevant at the same time.
- Temporal availability
- Locations or depots at the beginning and at the end of the planning horizon
  In operational planning, locations are given, whereas in tactical planning, it may be left to the solution algorithm to determine appropriate locations for the vehicles. In operational and tactical planning, locations at the end of the planning horizon may be arbitrary if open tours are allowed.
- Type and technical equipment:
  There are different criteria which determine whether or not a vehicle is in principle able to perform a request, disregarding the current point in time, location or capacity utilization:
  - Vehicle class (lorry, train, ship)
  - Vehicle type (swap-body vehicle, tank vehicle etc.)
  - Dimensions and weight
  - Technical equipment (equipment for dangerous goods, tail-lift, forklift on board etc.)
- Multiple compartments
  In tank vehicles, there are often several compartments which can be filled separately to allow the simultaneous transport of different goods or products or requests. When there are n compartments, n different products can be transported. When all compartments are used, no request can be executed that requires the transport of another good, even when none of the capacity constraints listed above would be violated.

Also the number of vehicles of each type and class is important. In reality, the number of vehicles is of course always limited. For tactical planning of the fleet size and mix, it may however be interesting to allow an unlimited number of vehicles of each type. This feature is, however, only supported by half of all systems.

Although the concrete class and type of a vehicle is irrelevant for a solution algorithm, the surrounding software must be able to manage it in order to give sensible feedback to the user. In this respect, most systems are capable of considering lorries/tractors, trailers/semi-trailers, and cars, but only few systems can also manage pedestrians, bicycles (which are both relevant, for example, in mail and newspaper delivery or meter reading), trains, ships, or aircraft. Similarly, the technical equipment of a vehicle is only relevant to determine vehicle-request compatibility, and the dimensions and weight of a vehicle are only relevant to determine vehicle-location compatibility.

3.3.2.2 Driver-related features

As far as drivers are concerned, restrictions regarding qualifications limit the compatibility between drivers and vehicles as well as drivers and requests. Such qualifications may be the type of driving licence a driver possesses, whether or not a training for the transport of dangerous goods was completed, or knowledge of customer or region specifics. A solution algorithm must be able to model the resulting compatibility constraints. About half of all systems offer this feature. However, as will be discussed below, in most cases there is no independent planning of vehicles and drivers, neither in academic publications nor in the real world. Usually, a vehicle and a driver are considered an inseparable unit. In many cases in practice, drivers are assigned manually to vehicles or vehicle routes after the latter have been planned.
Another matter of utmost importance in real-world lorry road transport are driver rules. In the European Union (EU) and in other parts of the world, there is extensive social legislation on driving, working, break and rest times for drivers; see Humphreys [32] for an overview. The automatic tachograph introduced in the EU nowadays allows for much tighter supervision, and the road transport industry in Europe is acknowledging that today, it has to comply with the regulations very exactly. Consequently, all CVRS manufacturers claim that their systems possess the feature of considering the pertinent legislation. During the study, however, the author has gained the impression that many systems do consider these rules only incompletely. In particular, only few systems, according to the study, contain the rules for double-manned vehicles.
It is important to note that a mathematical algorithm cannot determine a ‘legal schedule’ for a tour, because the term ‘legal schedule’ has no legally binding mathematical definition; it is a purely juristic concept. In an unlucky attempt to provide flexibility in practice, the European Union has introduced an intractably complex set of optional rules along with the mandatory ones. These rules leave a lot of room for interpretation, and a dispute about the legality of a schedule will eventually have to be settled in court. For practitioners, this means that trying to exploit the optional rules is dangerous. For algorithm developers, the optional rules mean a lot of tedious work: On the one hand, for considering them, on the other hand, for ensuring that the overall algorithm is not slowed down too much. Recently, there has been an increasing number of academic publications on driver rules (see, for example, Goel [25], Drexl/Prescott-Gagnon [16], Goel [26], Kok et al. [35], Prescott-Gagnon et al. [47]). The algorithms presented in these papers show the distressing complexity of driver rules.

3.3.2.3 Location-related features

The most important feature with respect to the locations to be visited are of course time windows. The large majority of systems supports multiple time windows. Vehicle-dependent time windows at locations (for example, large delivery vehicles having more restrictive access to customers in inner-city zones than small ones) are only rarely supported though.
In addition, general vehicle-dependent reachability constraints due to vehicle types or dimensions can be considered by more than half of all systems.

3.3.2.4 Road-network-related features

In this category, there are three important aspects. The first one is the usability of roads by certain vehicles or vehicle types. Depending on its size, weight or emission level, a vehicle may be unable or not allowed to use certain roads. This aspect has to be considered in the preprocessing phase, when distance and travel time matrices are computed. It is interesting to note that, for lorry transport, correctness of data is still an issue: The geographical data for lorry routing still does not cover all relevant attributes (passage heights, barred roads etc.) in a truly reliable fashion even in Western Europe. This is due to the fact that the commercial providers of geographical data have concentrated on the much larger car navigation system market. This, however, is about to change.

The second one is time-dependent travel time (longer travel time during rush hours), which is particularly relevant in inner-city transport. These times are mostly based on historical data. Two aspects have to be distinguished here: On the one hand, the method of determining the travel times, on the other hand, the way in which these times are considered in the planning algorithm.

The third one is that usability of roads may be dynamic when real-time road data on congestions, accidents etc. is taken into account.

Most systems cover the first aspect in their routines for computing distance and travel time matrices, but do not yet include the second or third aspect.

3.3.2.5 Request-related features

There is a large number of aspects of requests that (may) have to be taken into account to solve practical problems.

First of all, time windows are central properties of requests. Similar to the case for locations, most systems can handle single as well as multiple time windows for requests, and vehicle-dependent time windows are only seldom considered.

Another important aspect is precedence. The obvious 'pickup-before-delivery' precedence constraint can of course be handled by all systems. The requirement of request precedence within a route is also commonly covered. A nested execution (so-called LIFO loading: pickup request A, pickup request B, deliver B, deliver A) is supported by half of all systems. A feature that requires synchronization between different tours is precedence constraints of requests on different routes. This can have several causes, the most common one being transshipment of load. Only few systems can handle this.

Vehicle-driver-request compatibilities are the third fundamental aspect. Depending on the vehicle characteristics and the driver qualifications, not all requests may be performed by all vehicles and drivers, even if the request locations are accessible to the vehicle and the driver. This is considered by the large majority of systems.

Further practically relevant aspects of requests are:

- Optional requests (requests that need not be assigned to a route, but whose execution brings a bonus)
- Periodic requests (requests that have to be executed several times within a planning horizon, mostly according to visitation patterns, for example, twice a week, but not on consecutive days)
- Expected requests (requests that have not yet been issued by the customer, but which will probably be)
- Incompatible requests (parallel incompatibility: do not transport the requests at the same time with one vehicle; sequential incompatibility: do not transport request B on a route that has transported request A before)
- Indirect requests (for example, automatic generation of empty container balancing requests)
- Interdependent requests (requirement that two different vehicles are present at a location at the same time or with a fixed offset to perform a request)

Surprisingly, the above features are sorted by decreasing number of systems that support them, from two thirds to one quarter.

An aspect hardly ever found in the academic literature are so-called complex requests, which consist of more than one pickup and one delivery location. Almost half of all systems are able to handle such requests, which indicates that this is a common requirement in practice.

A further very difficult aspect is when there are different ways of performing a request. This issue arises regularly in freight forwarding companies and in multi-modal transport, where, for example, a load from A to B may be performed by a direct transport from A to B, via a meet-and-turn operation, or via one or several hubs. A different example is parcel delivery, where a package may be delivered to the recipient's office address from nine to five o'clock, and to his home address after six o'clock or on Saturdays. This raises the additional question of how to choose a way of performing the request (where to perform a meet-and-turn operation, when to deliver a package). Some systems contain features for handling special cases of such requests. In the academic literature, there are approaches for cases with transshipments, see below, but a generic unified concept is still missing.

3.3.2.6 Tour-related features

The standard case for tours is a closed loop, starting and ending at the same location (depot). Most systems, in addition, also support the planning of open tours, where the vehicle may be at any location at the end of its tour. Moreover, most systems also allow planning of multiple tours for one vehicle.

Further types of tour which are supported by most systems are

- Tours for lorries and trailers, where the trailer may be uncoupled and left behind at parking places, but with a fixed assignment of lorry and trailer
- Tours with special geographic or optical properties (consideration of fixed tour zones from tactical planning, intersection-free tours, 'good-looking' tours)
- Tours with a maximum waiting time

A common stipulation in practice is that all tours of a tour plan be similar with respect to covered distance, duration, number of requests, or costs. Almost half of all systems support this feature.

One very important aspect can be subsumed under the terms 'interdependent tours' or 'synchronized tours'. This refers to the fact that, for many different reasons, tours of different vehicles can depend on one another. The usual assumption in almost all vehicle routing algorithms is that the only coupling or linking or joint constraints between the tours of different vehicles are related to request covering, to ensuring that each request is performed exactly once. The following types of tour, however, require multiple synchronization of vehicles or tours with respect to space, time, load, or common scarce resources:

- Meet-and-turn tours with exchange of swap-body platforms
- Inter-tour resource constraints such as processing capacities at depots, maximum number of vehicles arriving at a depot per time unit due to limited number of ramps or conveyor belt capacities etc.
- Transshipments of load, that is, not only exchanging the complete load in a swap-body platform, but partial exchange or one-way transfer of load from one vehicle to another, for example in multi-modal transport
- Planning of separate tours for lorries/tractors and trailers/semi-trailers (free lorry-trailer assignment, trailers may be pulled by different lorries on their itinerary)
- Planning of separate tours or rotations for vehicles and drivers

Although ubiquitous in practice, these features are supported by few systems only. Mostly, these requirements are left to manual planning.

3.3.3 Objective functions

With respect to objective functions, almost everything that is reported in the literature is also available in all or most systems: It is possible to minimize the number of vehicles used, the overall distance covered by all vehicles and the total cost of all vehicles. In addition, about half of all systems support a weighted sum of one-dimensional objective functions, hierarchical, or multi-criteria objective functions.

3.3.4 Planning modes

The classical CVRP corresponds to an operational, single-period, static, deterministic planning situation. Given that reality is neither static nor deterministic, it is not surprising that CVRSs support different planning modes.

With respect to the frequency of planning, tactical planning of standard or base tours using aggregate, average data as well as operational, day-by-day planning is supported by all systems. Some CVRSs use different algorithms for these two modes, taking into account that for tactical planning, running time is not critical.

Multi-period or rolling horizon planning is also supported by most systems.

In addition, the possibility for dynamic or real-time planning is offered by most systems (changing assignments of requests to vehicles while the latter are already en route, caused by events such as new requests or breakdowns of vehicles).

Finally, interactive planning is supported in most cases. This means that the user can make small changes to an existing plan proposed by the algorithm, such as fix assignments of requests to tours, fix the sequence of partial tours, manually assign a certain vehicle to a tour etc. Both features require the capability to re-optimize an existing plan after small changes, without changing the fixed parts.

A related issue is stochastic data. This means that relevant information is given by a probability distribution instead of by a concrete number. Only 3 companies state that their algorithms are capable of handling stochastic customers (necessity to visit a location is stochastic) or stochastic demand/supply (amount of demand/supply is stochastic).

3.4 Algorithmic features

Whereas participating in the study was seen as a marketing measure by most CVRS manufacturers, several firms were reluctant to specify details about the algorithms used in their software. However, the questions in this part of the questionnaire were still answered by 21 firms, so that also these results may be considered representative.

Of course, any algorithm used for vehicle routing in practice is necessarily a heuristic. Therefore, the questions on algorithms asked which construction and improvement procedures and which metaheuristics were used.

3.4.1 Constructive procedures

The ranking of constructive procedures is as follows (number of mentions in parentheses):

1. Parallel savings (16)
2. Insertion (11)
3. Cluster first, route second (10)
   Nearest Neighbour (10)
5. Proprietary (7)
   Sequential savings (7)
7. Dynamic programming (6)
8. GENI intra-tour (5)
9. Regret (4)
   Route first, cluster second (4)

3.4.2 Improvement procedures

The ranking of improvement procedures is as follows:
1. Relocate (move a request to another tour) (16)
2. k-opt (15)
3. Swap/exchange (exchange two requests between two different tours) (13)
4. String-relocate (move a tour segment to another tour) (12)
5. Cross/string-exchange (exchange two tour segments between two different tours) (11)
6. Or-opt (8)
7. k-opt* (generalization of k-opt to capacitated problems) (7)
8. Lin-Kernighan (6)
9. GENI inter-tour (5)
   λ-interchange (exchange at most λ requests between two tours) (5)
   (Very) Large-scale neighbourhood search ((V)LSNS, exponential-size neighbourhoods) (5)
12. Double-bridge move (3)
    Ejection chains/cyclic transfers (move a fixed number of requests from tour 1 to tour 2, then the same number from tour 2 to tour 3 etc.) (3)
14. Proprietary (2)

3.4.3 Metaheuristics

The ranking of metaheuristics is as follows:
1. Tabu search (10)
2. Genetic algorithms (8)
3. Threshold accepting (7)
4. Proprietary (6)
   Ruin-and-recreate/fix-and-optimize/ripup-and-reroute (6)
   Simulated annealing (6)
7. Adaptive large neighbourhood search (5)
8. Ant colony systems (4)
   Guided local search (4)
   Variable neighbourhood descent (4)
   Variable neighbourhood search (4)
12. Greedy randomized adaptive search procedure (3)
    Memetic algorithms (3)
14. Adaptive guided evolution strategies (2)
    Attribute-based hill climber (2)
    Backbone search (2)
    Great deluge (2)
    Indirect search (decoder) (2)
    Neural networks (2)
    Scatter search (2)
21. Adaptive/approximate dynamic programming (1)
    Adaptive memory programming (1)
    Artificial immune systems (1)
    Particle swarm optimization (1)
    Record-to-record travel (1)

3.4.4 Mathematical-programming based approaches

The ranking of mathematical-programming based approaches is as follows:
1. Branch-and-cut (5)
   Constraint programming (5)

3. Column generation (4)
4. Branch-and-price (3)
5. Benders decomposition (1)
   Lagrangian relaxation (1)

3.4.5 Components, libraries and benchmarks used

The ranking of solvers, programming frameworks and algorithm libraries is as follows:
1. CPLEX (3)
2. Boost (2)
   COIN (2)
   LEDA (2)
   XPRESS (2)
6. BCP (1)
   CBC (1)
   Gurobi (1)
   lp_solve (1)
   SCIP (1)
   SoPlex (1)

8 firms stated that they had tested their algorithms with the Solomon VRPTW benchmarks, 6 have used the Gehring/Homberger VRPTW problems, and 2 the Li/Lim PDPTW instances. No firm was willing to tell anything about the results.

4 The gap between theory and practice

It goes without saying that CVRSs are commercial products for end users without programming and OR skills. Therefore, they have to offer an up-to-date GUI with adaptable look-and-feel as well as a help system. Moreover, a comfortable interface to common TMS or ERP systems is also an essential feature. (Together with the usual GIS, telematics, and statistics modules, this means that, typically, less than 10 % of the code of a CVRS are for the VRP solution algorithms.) Most importantly, however, academic codes usually work for special, mostly idealized, types of VRP only. Finding new best solutions for benchmark problems is regarded as a sport. CVRSs, on the other hand, must be able to handle many different problems. It is not an option to develop and implement a specialized algorithm for each new customer. Therefore, algorithms used in CVRSs must necessarily be generic and easily extendable to new problem features. Summing up the previous section, an algorithm for use in a state-of-the-art CVRS supports the following features:

- Pickup-and-delivery requests
- Multiple use of vehicles
- Multiple time windows for locations and requests
- Compatibility between locations, requests, vehicles and drivers
- Consideration of service times
- Fixed, distance-, time-, stop-dependent, penalty costs, tariffs, weighted, hierarchical cost functions
- Multiple capacity constraints
- Heterogeneous fleet with respect to cost, capacity, start and end depots
- Driver rules
- Dynamic planning over one-week planning horizon with event- or time-based rolling horizon planning
- Re-optimization options
- Interactive planning

Few algorithms described in the literature, if any, are able to deal with all these features.

A further difference is that for benchmarking, computation times do not matter. It is legitimate to run a code on one instance for days. In practice, computation time is obviously critical for operational and real-time planning, although less so for tactical planning. Moreover, the instances that must regularly be solved in practice are much larger (with respect to the number of requests) than standard benchmark instances.

The central point in the above-mentioned paper by Sörensen et al. [51] is that, according to these authors' experience, commercial CVRSs use quite a large number of local search improvement heuristics, based around different neighbourhoods, to improve initial solutions determined by constructive procedures. This is in contrast to academic codes which rather use complex, sophisticated techniques, but few neighbourhoods. The reasons for this are, according to Sörensen et al., that (i) an approach using many neighbourhoods may overcome the greedy behaviour of one that uses only a single neighbourhood, and (ii) supplying a large arsenal of local search strategies based on different neighbourhoods allows a flexible adaptation of the software to the specific requirements of each customer. The results of the study described in the present paper support this observation: As can be seen in Section 3.4.2, 27 codes use 111 different improvement procedures. This would mean an average of more than four even if no procedure were used by more than one vendor. An analysis of the detailed results shows that, in fact, the average is 6.75.

Aspects which are rather well-studied in theory but have not yet found widespread use in CVRSs and where, consequently, there is an application gap, are:

- Stochastic vehicle routing (see the surveys by Flatberg et al. [18] and Cordeau et al. [10])
- Time-dependent travel times (see Fleischmann et al. [19], Taniguchi/Shimamoto [52], Haghani /Jung [29])
- Mathematical-programming based approaches (see the monograph on matheuristics edited by Maniezzo et al. [39])

On the other hand, a research gap is apparent with respect to the following aspects:

- There are only very few papers on problems with optional, expected, indirect, or complex requests. The existing literature is mostly concerned with deterministic problems where all requests must be fulfilled and consist of single visits or one pickup and one delivery. Exceptions to this rule are Røpke [49] and Goel/Gruhn [27].
- Soft constraints such as visual attractiveness of tours, preferred assignment of certain drivers or vehicles to customers or 'fair' and balanced sharing of the workload between different tours are seldom considered (Lu/Dessouky [37], Bredström/Rönnqvist [5]).
- Tariffs and complex cost functions are often used in practice, but rarely considered in the literature. See Ceselli et al. [6] for a striking exception.
- Practitioners need robust, fast, extensible and simple (parameter-free) algorithms capable of solving instances with thousands of requests. The last 0.1 percent in solution quality which are to be gained from an additional complex algorithmic device are insignificant. On the other hand, the academic literature is to a large extent focussed on improving best known results for benchmark instances or solving concrete real-world problems with prototypical implementations of specialized algorithms. This point is further elaborated in Cordeau et al. [9] and Pisinger/Røpke [44].

Most importantly, models and algorithms for integrated and synchronized vehicle routing are still scarce. The following quote taken from Irnich [33], p. 9, still holds:

'While research on integrated models and solution methods for combined vehicle and crew *scheduling* has made some remarkable advances . . . , the literature on integrated vehicle *routing* still mainly focuses on *location routing problems* and *inventory routing problems*. Literature on other forms of integration is scarce. There is a need for new and improved techniques to attack integrated planning problems. As far as we can see, there is no convincing concept for dealing with VRPs with load transfer at hubs or consolidation points, especially in the context of bimodal or multimodal traffic. The same is true for longhaul goods traffic, which requires the coordination between feeder processes, linehaul, and distribution.'

More specifically, Macharis/Bontekoning [38], p. 400, state in their survey of intermodal freight transport that 'intermodal freight transportation research is emerging as a new transportation research application field, that it still is in a pre-paradigmatic phase, and that it needs a different type of models than those applicated to uni-modal transport'.

In almost all vehicle routing models and algorithms, the tours of the different vehicles are assumed to be independent of one another, so that changes on one tour do not have any effects on other tours. However, in a surprisingly high number of cases, this assumption does not hold. Examples for practical applications requiring a spacial, temporal, and in some cases also load-related, synchronization or coordination of tours are:

- Planning of inter- and multi-modal transports
- Planning of meet-and-turn tours, transports over hubs or cross-docking locations
- Simultaneous planning of lorries/tractors and trailers/semi-trailers, if there is no fixed assignment of a trailer to a motor vehicle
- Simultaneous vehicle and crew routing, if there is no fixed assignment of a driver to a vehicle
- Automatic planning of multiple types of resources (drivers, lorries/tractors, trailers/semi-trailers, swap-bodies/containers)

The results of the study show that several CVRSs contain solutions to concrete problems in this area. A general, unifying modelling and solution concept is still missing; science has to catch up in this respect. This statement is supported by the fact that the survey article by Gendreau et al. [22], which presents an extensive literature list on VRPs, does not mention a reference on VRPs with multiple synchronization constraints. Similarly, the recent monograph by Golden et al. [28] does not contain a paper on this topic. This does not mean that there are no such papers at all. Rather, it shows that no systematic study of this problem class has yet been performed. Such research is now beginning to emerge (Drexl [15]). On the other hand, judging from this author's professional experience in the freight forwarding business, demand from practice is present in any case.

5 Conclusions and outlook

The exact solution of even the basic variants of VRP is still impossible for instances of realistic size. An exact solution of real-world problems with many additional side constraints will remain impossible in the short and medium term. However, close-to-optimal solutions of more and more complex and integrated problems, increasingly based on incomplete optimization approaches and mathematical-programming based heuristics, are possible, and this is sufficient to provide useful decision support in practice.

For the foreseeable future, CVRSs will remain decision *support* systems in almost all application areas. Essentially, fully automatic planning is possible only in some special cases, most notably in intra-plant logistics. In road and inter-modal transport, interactive planning with a human dispatcher having the final say is and will remain the rule. A modern CVRS, however, can considerably facilitate the daily routine work for human decision-makers. The systems have become so mature and user-friendly that, nowadays, after introducing a CVRS, nobody wants to return to purely manual planning any more. The concerns often voiced by many dispatchers, CVRSs would invalidate their knowledge and experience or even make them lose their jobs, are unfounded. This has become evident in many discussions with manufacturers as well as users of CVRSs.

Summing up, CVRSs constitute a fixed and indispensable component of logistics planning in practice. Just like any other product, CVRSs have to adapt to ever-changing customer needs and expectations. This requires constant further development, both with respect to information technology and to OR models and algorithms. Consequently, even more than half a century after the first OR paper in this field, the practice of vehicle routing will continue to provide interesting and challenging problems for OR researchers.

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